Methods of finding Z, M, and E for events in CRIS and SIS

ACE Memo ID: 1997.07.10.BLD.a1 (JPL/C+S/ZMEalgorithms)

From: Brian L. Dougherty (bld@heag1.jpl.nasa.gov)

To: CRIS/SIS Data Analysis Teams

Date: 10 Jul. 1997

In this memo I present some thoughts on finding (Z,M,E) solutions for particle events in CRIS and SIS.

Suppose an unknown isotope enters a telescope, depositing energies E_i in detectors i = 1...n. We wish to identify this isotope, and measure its total incident energy. In general, the items we have to work with, and our goals, etc. are outlined below.

Geometrical inputs:

L_i : total thickness of each detector, mapped across its entire surface

δL_i : dead layer thickness of each detector, also mapped

Givens:

R(E;Z,M) : range-energy function/table for all isotopes

E(R;Z,M) : inverse of the above

Event-parameters:

(θ,ϕ,x,y) : particle trajectory, obtained from the hodoscope

Variables:

E_i : the deposited energies

Estimables:

∂E_i : fluctuations in energy loss (Landau, multiple scattering, etc.)

Unknowns to solve for:

(Z,M) : the isotopic charge and mass

$E = \sum_i (E_i + \delta E_i)$: the total energy incident on the telescope

where δE_i is the energy "missing" in the dead layers

For the sake of argument here, let us presume:

- Dead layers are at the exit face of each detector.
- Since ∂E_i fluctuations dominate the goodness of any (Z,M,E) solution, let us suppose the geometry and R-E functions are known with certainty.

Desiderata:

masses are real-valued

results are not presented as integer M's together with degrees of confidence

instead, our isotopic precision is displayed via mass distributions

precision and accuracy

detector thickness and dead layer maps are used

exact expressions are used where possible, not approximations

maximum use is made of the available event information

we want a measure (m) of how good the solution is

a complete solution includes (Z,M,E,m)

flags can be set to warn against multiple particles, breakup, hard collisions...

generalizability and flexibility

to treat penetrating particles

to avoid dead or noisy detectors

speed

fast search algorithms are sought

many events are batch processed at once, not one-by-one

A large class of traditional, ΔE-E' methods may be characterized thusly. We rewrite $\Delta E \equiv E_{n-1}$, $E' \equiv E_n$, $\Delta L \equiv L_{n-1}$, and $\delta L \equiv \delta L_{n-1}$. Then, since $\delta E_{n-1} = E[R(E') + \delta L \sec\theta] - E'$, we can eliminate $\delta E \equiv \delta E_{n-1}$ and obtain the following equations of merit, written only using known or measured inputs:

(1) $\Delta L \sec\theta = R\{\Delta E + E[R(E') + \delta L \sec\theta]\} - R(E')$

(2) $\Delta E = E[R(E') + \Delta L \sec\theta] - E[R(E') + \delta L \sec\theta]$

We may treat the right-hand sides of (1) or (2) as calculations. Results are to be compared against the known left-hand sides. Our aim is to find (Z,M) so that the equation of choice is solved most accurately.

We note that terms involving E' appear twice in each of these equations, making it hard to write a simple, alternative expression in which E' is isolated on the left-hand side. Also, both (1) and (2) involve two inversions of the R-E relation; in (1) these inversions are in one large term, but in (2) they are split between the two terms. We do not consider a more symmetrical equation in which each input occurs twice...

$\Delta L \sec\theta + R\{E[R(E') + \Delta L \sec\theta] - \Delta E\} = \delta L \sec\theta + R\{E[R(E') + \delta L \sec\theta] + \Delta E\}$

because there are four inversions. We also avoid another exact equation ...

$\delta L \sec\theta = R\{E[R(E') + \Delta L \sec\theta] - \Delta E\} - R(E')$,

since it may involve taking the difference of two large quantities to calculate a small quantity. Finally, only the R-E relation is used, thus avoiding any errors that may result from discrepancies between that and any (independently computed?) stopping power vs E terms. Otherwise we could use $\delta E \approx \delta L \sec\theta S(E')$ for $E' >> \delta E$, where $S = dE/dx$, and calculate $\Delta L \sec\theta \approx R(E' + \Delta E) - R(E') + \delta L \sec\theta S(E')/S(E' + \Delta E)$.

There are several ways of implementing these methods, and they generally return solutions that are as precise as any other. These methods fall short, however, with regard to a few desiderata. The total incident energy is not immediately calculated, and neither is a measure of goodness found. These shortcomings are correctable by simple augmentations. A more serious deficiency is that information is not used maximally. Also, a different algorithm must be invented for penetrating events, and to treat dead or noisy detectors.

A more direct approach, and one that satisfies our desiderata, is to treat the deposited energies, E_i as separate measures for the same event; i.e. as variables to be fitted by adjusting the parameters (Z,M,E) used in the known geometrical and R-E relationships. The equation of merit for this approach is:
$E_{ifit} = E[R(E) - \sum L_{i-1}\sec\theta] - E[R(E) - (\sum L_i - \delta L_i)\sec\theta]$, where $\sum L_i \equiv L_1 + L_2 \ldots + L_i$, and we define $E[x < 0] = 0$. For energy deposited in any particular detector, this equation alone suffices to select viable (Z,M) and E combinations. By comparing the results with other detectors, more restrictive choices of (Z,M,E) are made. The natural figure of goodness is the chi-squared, $\chi^2 = \sum_i (E_i - E_{ifit})^2/\partial E_i^2$ which is minimized when the best solution is found; i.e. it can become an integral part of the method. Any set of detectors can be used to make these comparisons, which allows us to utilize all the properly weighted information: dead detectors are simply eliminated in the sum, and ∂E is increased to account for noisy detectors. The total incident energy, E may be searched for, rather than calculated, so penetrating and stopping events could be handled in essentially the same way. Of course, when a particle is known to stop in the n^{th} detector, then our equation $E_{nfit} = E[R(E) - \sum L_{n-1}\sec\theta]$ has so much "leverage" that rapid convergence and precise solutions are possible.

Implementation is just as variable as for the traditional methods. The rest of this memo will be spent describing some of the algorithms I've tried.

First of all however, let me not generalize to penetrating events yet, but consider only stopping events. A better comparison with traditional methods may then be made.

I happen to use the Hubert, Bimbot, Gauvin (HBG) range-energy tables, which give R vs E for all Z's. They consider only a single representative M for each Z. Their ranges are integrated along the pathlength, not projected. I interpolate in log-log space for intermediate values.

Charge is found by first summing the recorded energies, $E_{sum} = \sum_i E_i$, and assuming that this is a good approximation to the total incident energy. In other words, I neglect the small δE's. I then loop over all possibilities of charge ($1 \le Z \le 30$) to compute E_{ifit} for each detector. I use the HBG representative M's in these calculations. That Z which returns the lowest χ^2 is chosen as the correct value. Refinements are possible, of course, but this algorithm seems to work satisfactorily.

With charge known, mass and energy may be found many different ways. But there is no easy means to handle M and E simultaneously. Instead, the general approach is to perform some "outer" search of mass hypotheses, and compute an "appropriate" energy each time.

One straightforward method is simply to loop over all reasonable hypotheses of M: say from M(HBG) – 5 AMU to M(HBG) + 5 AMU, in 1 AMU steps. Then nest a loop over all reasonable values of E: say from Esum – 10 MeV to Esum + 50 MeV, in 3 MeV steps. An inital approximation of mass comes from that (M_1,E) combination which returns the lowest χ^2. Next, loop over M from M_1 – 1 AMU to M_1 + 1 AMU in 0.1 AMU steps, and nest the same energy loop as before. A final (M,E) solution once again gives the lowest χ^2. This brute force algorithm works OK, finding the best solutions when available, but it is terribly slow. In all, there are 32 mass guesses and 21 energies; i.e. 672 calculations of χ^2 are performed. Of course, the range and coarseness of these searches can be adjusted, but a different approach is needed to really improve speed.

Two alternative energy calculations have been tried. Each dramatically reduces the number of times data are reused. But each also veers away from constant emphasis on χ^2. Instead, they attempt initially to derive the correct total energy, given the mass hypothesis currently under consideration. The fastest of these passes through the data only once, and starts by calculating the energy lost in the dead layer just above the stopping detector: $\delta E_{n-1} = E[R(E') + \delta L_{n-1}\sec\theta] - E'$, where $E' \equiv E_n$. Then the residual energy is reassigned as $E' \equiv E_{n-1} + dE_{n-1} + E_n$, and the process is repeated for the dead layer above the $(n-1)^{th}$ detector. At the top of the stack, a putative value for the total energy E_{calc} is returned, and

this is then used to re-compute E_{ifit} through the stack in order to construct the χ^2 for that mass hypothesis. It is presumed that the correct mass would lead to both $E_{calc} - E$ and χ^2 being almost minimized.

A binary mass search has been tried but ran into difficulties owing to the slightly asymmetrical shape of the χ^2 vs M curve, where χ^2 is computed after the energy derivation is made. A more successful alternative is to fit a parabola to three widely-spaced points on this curve. The derived minimum provides a first approximation for M, about which another parabola is fitted for three more closely spaced points. The second minimum is a good refinement, and a third such pass is even better. Thus, only 9 mass hypotheses are sampled, instead of 32 in the survey-method recounted above. But since the χ^2 vs M curve is not quite parabolic, the results are not as precise as those obtained by the survey. Some combination of these methods may be attempted.

I intend to do some comparisons between traditional and new methods of (Z,M,E) analyses. These will look at speed, precision, and extendability.