INTERNAL REPORT NO. 10

LOCAL TIMES USED IN MAGNETIC COORDINATE SYSTEMS

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Three local time systems will be discussed:

- Dipole Local Time (DLT) (Stone, 1964)
- Magnetic Local Time (MLT) (Fritz & Gurnett, 1965)
- Magnetic Equatorial Time (MET) (This report)

For a given dipole longitude Ψ (degrees):

(1) $$\begin{aligned} DLT(\Psi) &= DLT(0^{o}) + \Psi/15 \\ &= UT + (289.8/15) + \Psi/15 \end{aligned}$$

where UT = universal time (GMT) in hours, and 289.8^{o} is the geographic longitude of the north magnetic pole.

DLT is essentially the difference in <u>dipole</u> longitude between the spacecraft and the "magnetic prime meridian" (289.8° E. longitude) plus the difference in <u>geocentric</u> longitude between the m.p.m. and the sun.

MLT is essentially the difference in <u>dipole</u> longitude between the spacecraft and the sun.

(2) $$MLT = \pm (12/\pi) \text{Arccos} \left\{ \frac{(\vec{S} \times \vec{m}) \cdot (\vec{P} \times \vec{m})}{|\vec{S} \times \vec{m}||\vec{P} \times \vec{m}|} \right\} + 12$$ (Note 1)

where $\vec{S}$ is the unit vector pointing to the sun;
$\vec{P}$ is the unit vector pointing to the spacecraft;
$\vec{m}$ is the unit vector pointing north along the dipole axis

MET is similar to MLT, with the vector $\vec{P}$ replaced by $\vec{E}$, the unit vector pointing to the magnetic equator of the <u>real-field</u> line of force on which the spacecraft is located:

(3) $$MET = \pm (12/\pi) \text{Arccos} \left\{ \frac{(\vec{S} \times \vec{m}) \cdot (\vec{E} \times \vec{m})}{|\vec{S} \times \vec{m}||\vec{E} \times \vec{m}|} \right\} + 12$$

Note 1: The sign must be chosen appropriately in equations 2-4. This problem is not present in equations 1, 5, and 6.

(For comparison of the vector notations, DLT is given by:

$$(4)\quad DLT = \pm\,(12/\pi)\mathrm{Arccos}\left\{ \frac{|\vec{P}\vec{m}\vec{k}|\,|\vec{S}\vec{m}\vec{k}| + \left[(\vec{S}\times\vec{k})\cdot(\vec{m}\times\vec{k})\right]\left[(\vec{P}\times\vec{m})\cdot(\vec{k}\times\vec{m})\right]}{|\vec{S}\times\vec{k}|\,|\vec{P}\times\vec{m}|\,|\vec{k}\times\vec{m}|^2} \right\} + 12$$

where $\vec{P}\vec{m}\vec{k} = \vec{P}\cdot(\vec{m}\times\vec{k}) = (\vec{P}\times\vec{m})\cdot\vec{k}$, etc.
and $\vec{k}$ is the unit vector pointing north along the earth's rotational axis)

The vector representations are usually not convenient for calculation, especially for DLT, for which the first formula given (1) is quite easy to handle. Analogous formulas for MLT and MET are:

$$(5)\qquad MLT = (\Phi_0 - \Phi_s)/15. + 12.$$

$$(6)\qquad MET = MLT + \Delta/15.$$

where Φ_0 = dipole longitude of spacecraft (degrees);
Φ_s = dipole longitude of sun (degrees);
Δ = difference in dipole longitude between equator of real-field line and spacecraft position (degrees).

The dipole longitude of the sun can be obtained from the geocentric coordinates by an orthogonal transformation. The geocentric longitude of the sun is easily obtained from the universal time:

$$(7)\qquad \phi_s = 180(1 - T/43200)$$

where T = universal time (UT), in seconds;
ϕ_s = geocentric longitude of sun, in degrees.

The geocentric latitude of the sun (λ_s) is given by the following (Smart, 1944):

$$(8)\qquad \sin\lambda_s = (\sin\varepsilon)\left[\sin 2\pi\left(\frac{D-V}{365.24}\right)\right],$$

where $\varepsilon = 23.445^{\circ}$ = inclination of rotational axis;
D = day of year;
V = day number of vernal equinox.

The dipole longitude of the sun is then obtained from these values using an orthogonal transformation into the dipole system defined by placing the north dipole axis at geocentric coordinates 78.5°N, 289.8°E:

$$(9)\quad \begin{cases} S_x = \sin\Theta_s \cos(\phi_s + 70.2^\circ) \\ S_y = \sin\Theta_s \sin(\phi_s + 70.2^\circ) \\ S_z = \cos\Theta_s \end{cases}$$

$$(10)\quad \begin{pmatrix} S_x' \\ S_y' \\ S_z' \end{pmatrix} = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

$$(11)\quad \Phi_s = (180/\pi)\arctan(S_y'/S_x') \qquad \text{(Note 2)}$$

where $\Theta_s = 90^\circ - \lambda_s$;

$\alpha = 11.5^\circ$ = tilt of dipole axis;

S = unit vector to sun (geocentric coordinates);

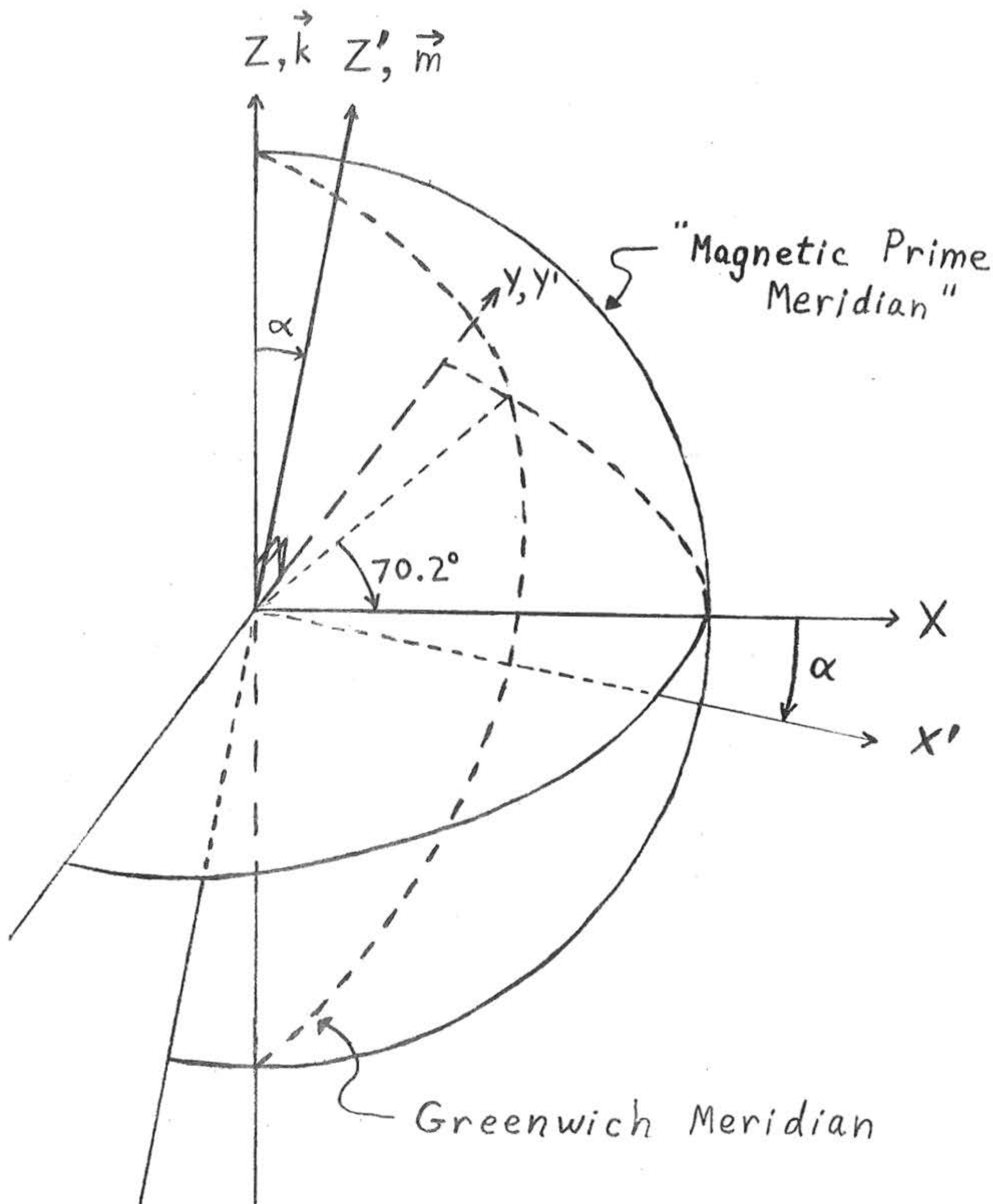
S' = unit vector to sun (dipole coordinates)

Computer programs are available (see writeups for METCON and METG/METD in 211 Downs) to compute Δ, MLT, and MET. Subroutine METCON performs the calculations (7) and (8), calls subroutine DICON to perform the transformation (9), (10), and (11), and then calculates MET using (5) and (6). MLT is obtained by setting $\Delta = 0$. If MET is desired, subroutine METG (or METD) first calculates Δ by interpolating between values calculated from the GSFC (12/66) field model at 10° intervals in dipole longitude and 5° intervals in dipole latitude for

$$60^\circ \leq |\text{ dipole latitude }| \leq 85^\circ.$$

The variations with altitude are included in the calculation, and are reasonably accurate for $0 \leq$ altitude ≤ 1400 km.

Note 2: Proper quadrant is chosen by FORTRAN function ATAN2(SY,SX).


$Z, \vec{k}$
$Z', \vec{m}$
"Magnetic Prime Meridian"
y, y'
α
$70.2°$
X
α
X'
Greenwich Meridian

Bibliography

Fritz, T. A., and Gurnett, D. A., "Diurnal and Latitudinal Effects Observed for 10 kev Electrons at Low Satellite Altitudes," J. Geophys. Res., 70, 2485-2502, 1965.

Smart, W. M., Textbook on Spherical Astronomy, Cambridge (Eng.), University Press, 1944.

Stone, E.C., "Local Time Dependence of the Non-Stormer Cutoff for 1.5 Mev Protons in the Quiet Geomagnetic Field," J. Geophys. Res., 69, 3577-3582, 1964.

Also see (same title) Enrico Fermi Institute of Nuclear Studies preprint number EFINS 63-79.