Droop in MAST matrix detectors

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revised 3/25/92

In this report I calculate the droop effect in the MAST matrix detectors. Droop refers to a deficit in the signal from the amplifiers due to the finite charge collection time from the detector combined with the amplifier shaping time. It depends on the details of the detector and amplifiers.

A simplified equivalent circuit for the detector and amplifiers is shown in Figure 1. The calculations based on this circuit are exact, but they are detailed enough that the results cannot be expressed in closed form and must be evaluated by computer.

I will first calculate the current as a function of time, $I(t)$, that leaves the detector and enters the first amplifier stage, due to a unit charge being collected in strip i at time $t = 0$. I assume that the charge is equivalent to a current source in parallel with the strip capacitor and that the collection time is short enough that the source current can be represented by a δ function in time.

The voltages, V_j, at the top of each strip labeled by j satisfy the set of circuit equations

$$-V_{j-1} + 2V_j - V_{j+1} + RC_j \frac{dV_j}{dt} = R\,\delta(t)\delta_{i,j} \tag{1}$$

which is the finite-difference version of a diffusion equation. Note that the strips have different capacitances due to their different lengths. We are interested in j values from 2 to $N-1$, where $N = 92$ is the number of strips, because $V_1 = V_N = 0$. Defining new variables by

$$x_j = C_j^{1/2} V_j \tag{2}$$

the set of equations (1) can be rewritten as the matrix equation

$$\mathbf{A}\mathbf{C}^{-1/2}\mathbf{X} + R\,\mathbf{C}^{1/2}\frac{d\mathbf{X}}{dt} = R\,\delta(t)\mathbf{e}_i \tag{3}$$

where the x_j are elements of the column vector $\mathbf{X}$, C_j are diagonal elements of the diagonal matrix $\mathbf{C}$, $\mathbf{e}_i$ is a column vector with 1 in element i and 0 elsewhere, and $\mathbf{A}$ is a symmetric tridiagonal matrix with diagonal elements equal 2, super- and sub-diagonal elements equal -1. Equation (3) is easily solved for $\mathbf{X}(t)$ by standard linear algebra methods. The solution in matrix form is

$$\mathbf{X} = \mathbf{V}\mathbf{E}\mathbf{V}^T\mathbf{C}^{-1/2}\mathbf{e}_i\Theta(t) \tag{4}$$

where $\Theta(t)$ is the unit step function, $\mathbf{V}$ is a matrix whose columns are the eigenvectors of the matrix

$$\mathbf{B} = \frac{1}{R}\mathbf{C}^{-1/2}\mathbf{A}\mathbf{C}^{-1/2} \tag{5}$$

$\mathbf{E}$ is a diagonal matrix whose diagonal elements are $e^{-\lambda_j t}$, and the λ_j are the eigenvalues of $\mathbf{B}$. For detectors with only a few strips the eigenvectors and eigenvalues of $\mathbf{B}$ can be calculated explicitly. For large N they must be calculated by computer. This is easy to do because $\mathbf{B}$ is tridiagonal and symmetric due to the change of variables from V_j to x_j, so the eigenvalues are all real and positive. I used the Numerical Recipes subroutine TQLI.

The current out of the detector is given by

$$I(t) = \frac{1}{R}C_2^{-1/2}x_2(1 - \delta_{i,1}) + \delta(t)\delta_{i,1} \tag{6}$$

Substituting for x_2 from (4) this can be rewritten as

$$I(t) = \sum_{n=2}^{N-1} a_n e^{-\lambda_n t}\Theta(t)(1 - \delta_{i,1}) + \delta(t)\delta_{i,1} \tag{7}$$

where the constants

$$a_n = \frac{1}{R} C_2^{-1/2} V_{2,n} V_{i,n} C_i^{-1/2} \tag{8}$$

Plots of I(t) versus t are given for various locations i of the charge collecting strip in Figure 2.

The current out of the detector goes through the various amplifier stages and is converted to a voltage pulse height by the ADC. The total current from each end of the detector is equal to 1, but the amplifiers cannot collect all of this current in the finite shaping time, which leads to the droop. The simplified amplifier circuit shown in Figure 1 can be solved analytically for the delta function response, $V^{\delta}(t)$. I did the algebra with Mathematica and the result is

$$V^{\delta}(t) = \sum_{m=1}^{4} b_m e^{-\mu_m t}\Theta(t) \tag{9}$$

where

$$b_1 = \frac{\mu_1(\mu_1 - \mu_2 - \mu_5)}{C_1 C_3(-\mu_1 + \mu_2)(-\mu_1 + \mu_3)(-\mu_1 + \mu_4)R_4} \tag{10a}$$

$$b_2 = \frac{\mu_2\mu_5}{C_1 C_3(\mu_1 - \mu_2)(-\mu_2 + \mu_3)(\mu_2 - \mu_4)R_4} \tag{10b}$$

$$b_3 = \frac{\mu_3(\mu_2 - \mu_3 + \mu_5)}{C_1 C_3(\mu_2 - \mu_3)(-\mu_1 + \mu_3)(-\mu_3 + \mu_4)R_4} \tag{10c}$$

$$b_4 = \frac{\mu_4(\mu_2 - \mu_4 + \mu_5)}{C_1 C_3(\mu_1 - \mu_4)(\mu_2 - \mu_4)(-\mu_3 + \mu_4)R_4} \tag{10d}$$

$$\mu_1 = \frac{1}{R_1 C_1}, \quad \mu_2 = \frac{1}{R_2 C_2}, \quad \mu_3 = \frac{1}{R_3 C_3}, \quad \mu_4 = \frac{1}{R_4 C_{\cancel{8}\,4}}, \quad \mu_5 = \frac{1}{R_5 C_2} \tag{10e}$$

The amplifier response $V^{\delta}(t)$ is plotted versus t in Figure 3.

The convolution of $I(t)$ from (7) with $V^{\delta}(t)$ from (9) gives the final result for the voltage out of the amplifiers

$$V(t) = \sum_{n=2}^{N-1}\sum_{m=1}^{4} \frac{a_n b_m}{\mu_m - \lambda_n}(e^{-\lambda_n t} - e^{-\mu_m t})\Theta(t)(1 - \delta_{i,1}) + V^{\delta}(t)\delta_{i,1} \tag{11}$$

The total charge collected from both ends of the detector according to the output of the amplifiers is

$$Q_i = \frac{V_i^{\max} + V_{N-i+1}^{\max}}{V_1^{\max}} \tag{12}$$

where $V_i^{\max}$ is the maximum of $V(t)$ for an event in strip i, which I calculated with the Numerical Recipes golden search subroutine GOLDEN. The position determined from an event in strip i is

$$X_i = \frac{V_i^{\max}}{Q_i \, V_1^{max}} \tag{13}$$

while the true position of the event is

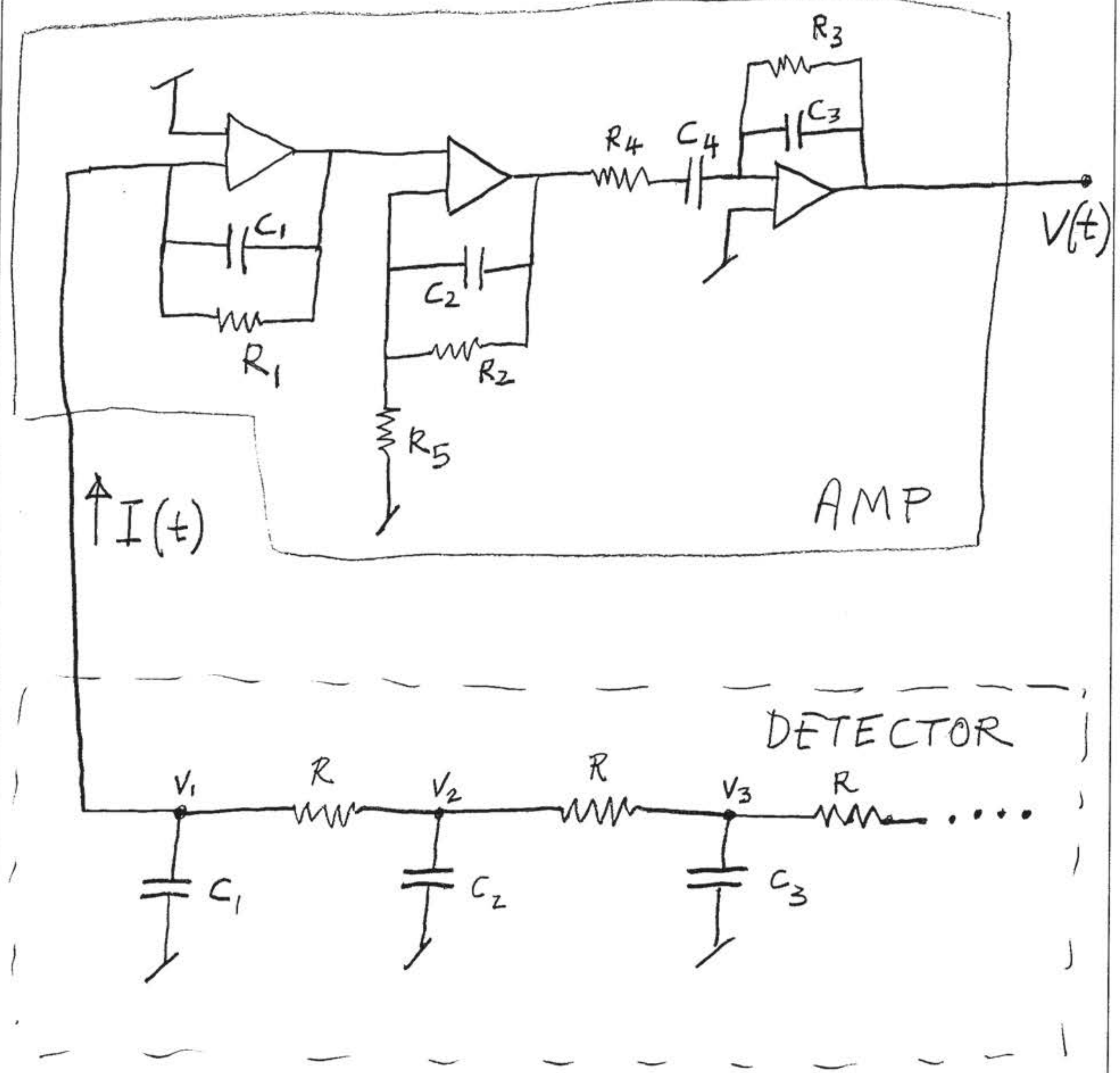
$$X_i^{true} = \frac{N - i}{N - 1} \tag{14}$$

The error in position due to the droop, normalized to the strip width, is

$$\Delta X_i = N(X_i - X_i^{true}) \tag{15}$$

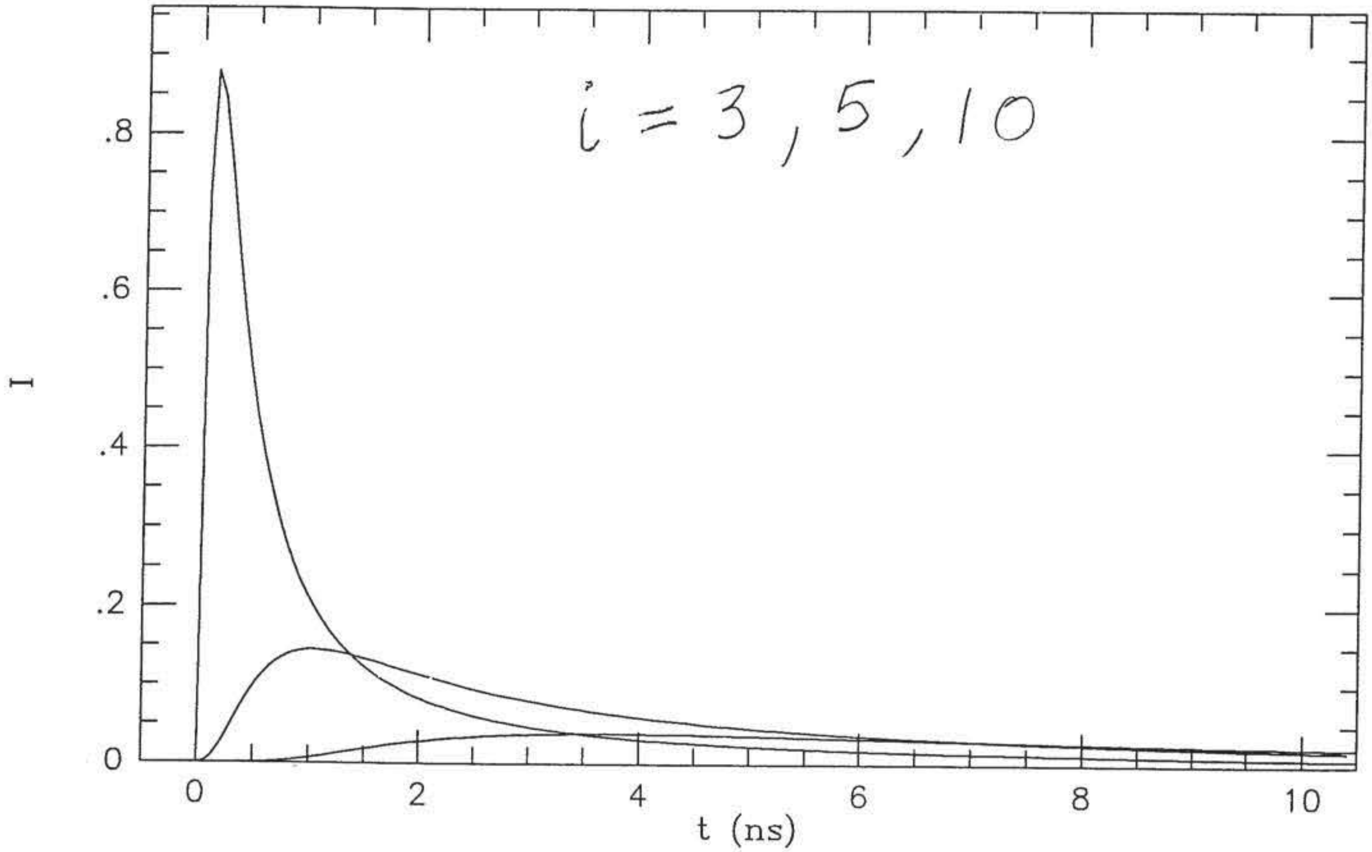
Q_i and ΔX_i are plotted versus i in Figure 4. The droop is seen to produce a maximum charge deficit of ~12% and a maximum position error of ~0.8 strip widths. Some real data for Q_i are shown for comparison in Figure 5. A table of i, Q_i, X_i, X_i^{true}, and ΔX_i, which can be used to correct data from MAST, is also attached. This only needs to be recalculated if the equivalent circuit is modified.

Figure 1

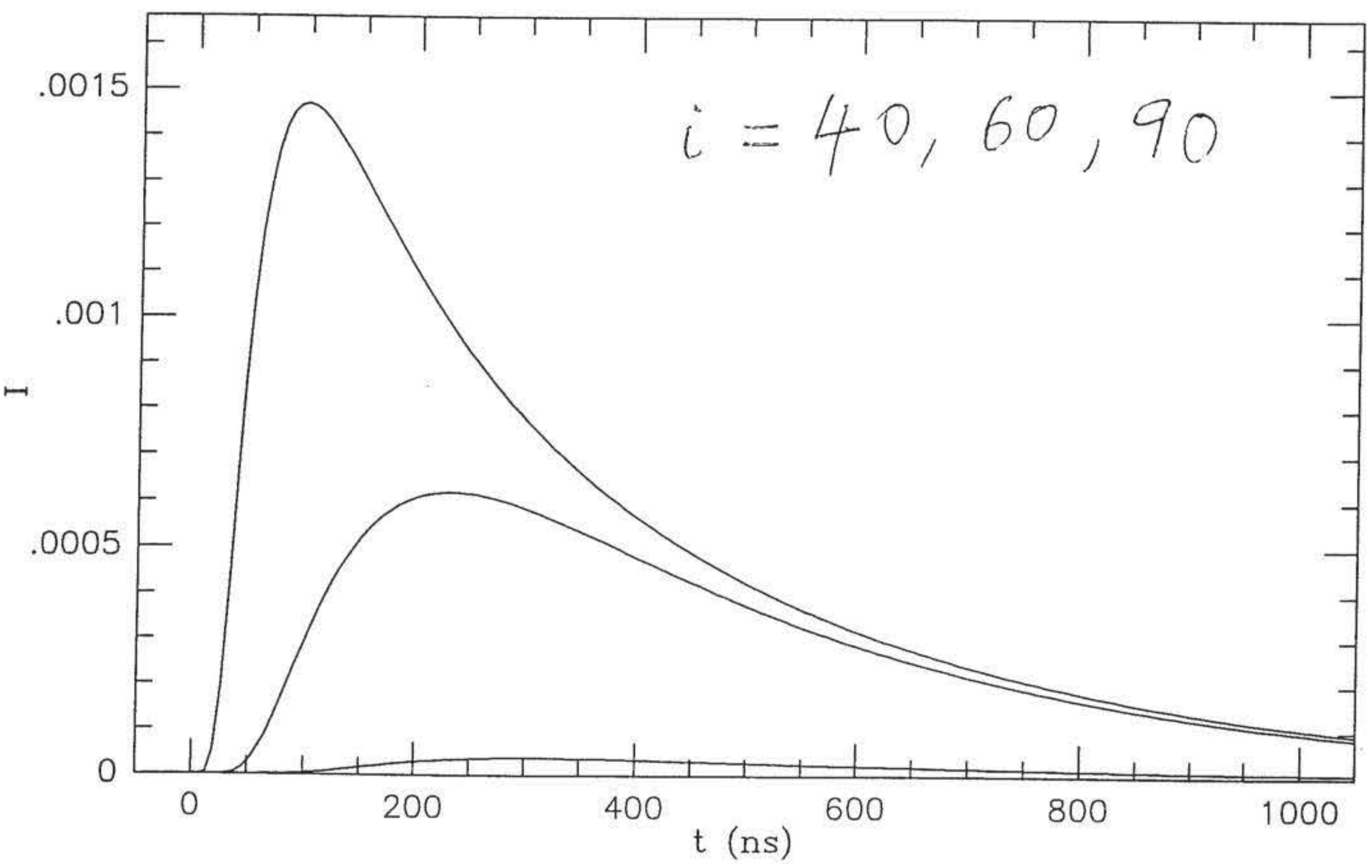


Amp:	Detector:
$R_1 = 7.5K$	$R = 19.1\,\Omega$, $N = 92$
$R_2 = 13.3k$	$C_d = 1840\,pF$
$R_3 = 8.67k$	$C_j = C_d \dfrac{l_j}{\sum_k l_k}$
$R_4 = 1.35K$	
$R_5 = 1.0\,k$	strip length, $l_j = \left(b_j[2r_d - b_j]\right)^{1/2}$
$C_1 = 120\,pF$	
$C_2 = 60\,pF$	$b_j = b_0 + (j-\frac{1}{2})W$, $r_d = 2.525$ cm
$C_3 = 100\,pF$	$W = 2\dfrac{(r_d - b_0)}{N}$, $b_0 = 0.245$ cm
$C_4 = 600\,pF$	

Figure 2



i = 3, 5, 10
.8
.6
.4
.2
0
I
0
2
4
6
8
10
t (ns)



i = 40, 60, 90
.0015
.001
.0005
0
I
0
200
400
600
800
1000
t (ns)

Figure 3

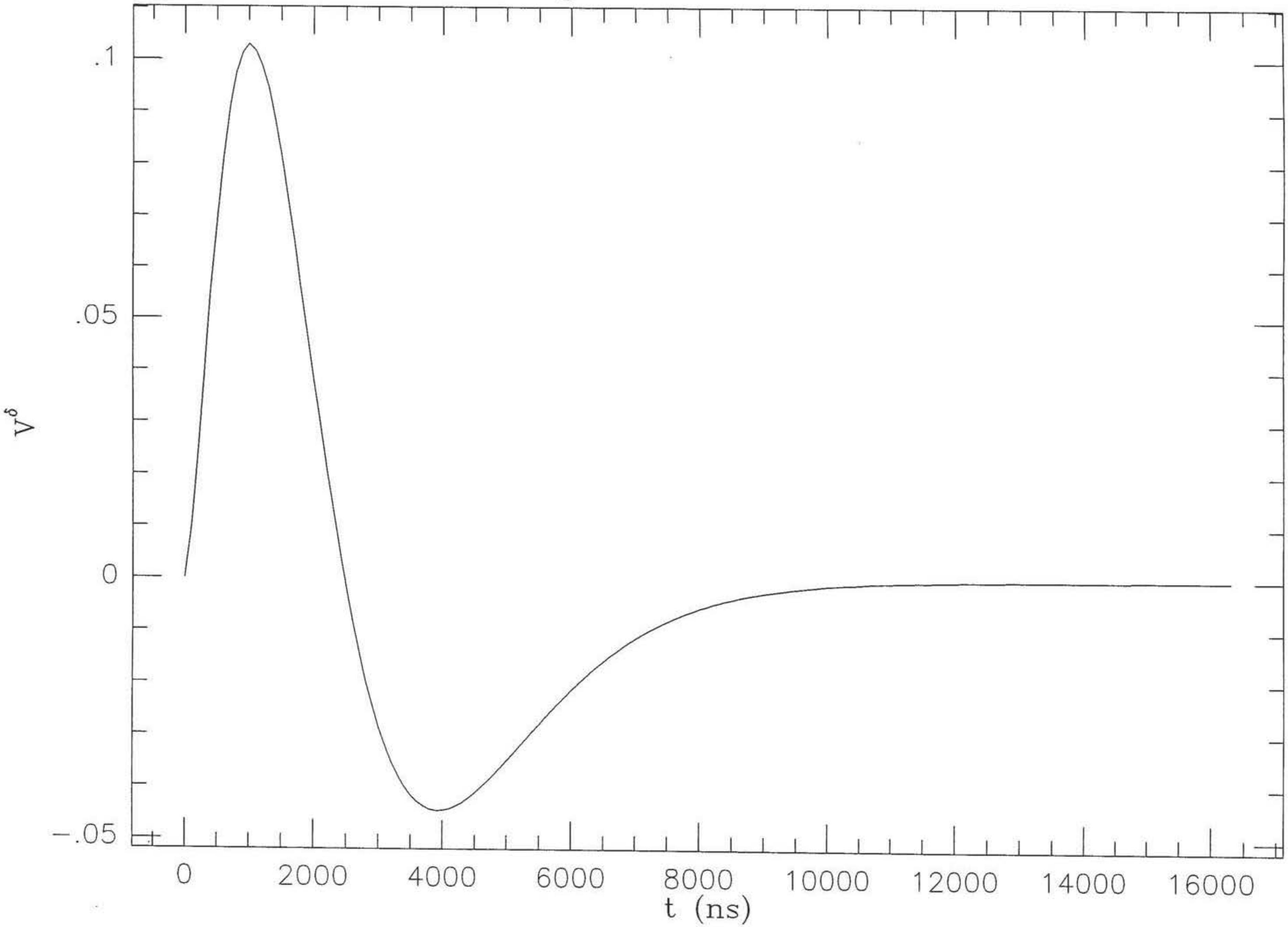

.1
.05
0
−.05
V^δ
0
2000
4000
6000
8000
10000
12000
14000
16000
t (ns)

Figure 4

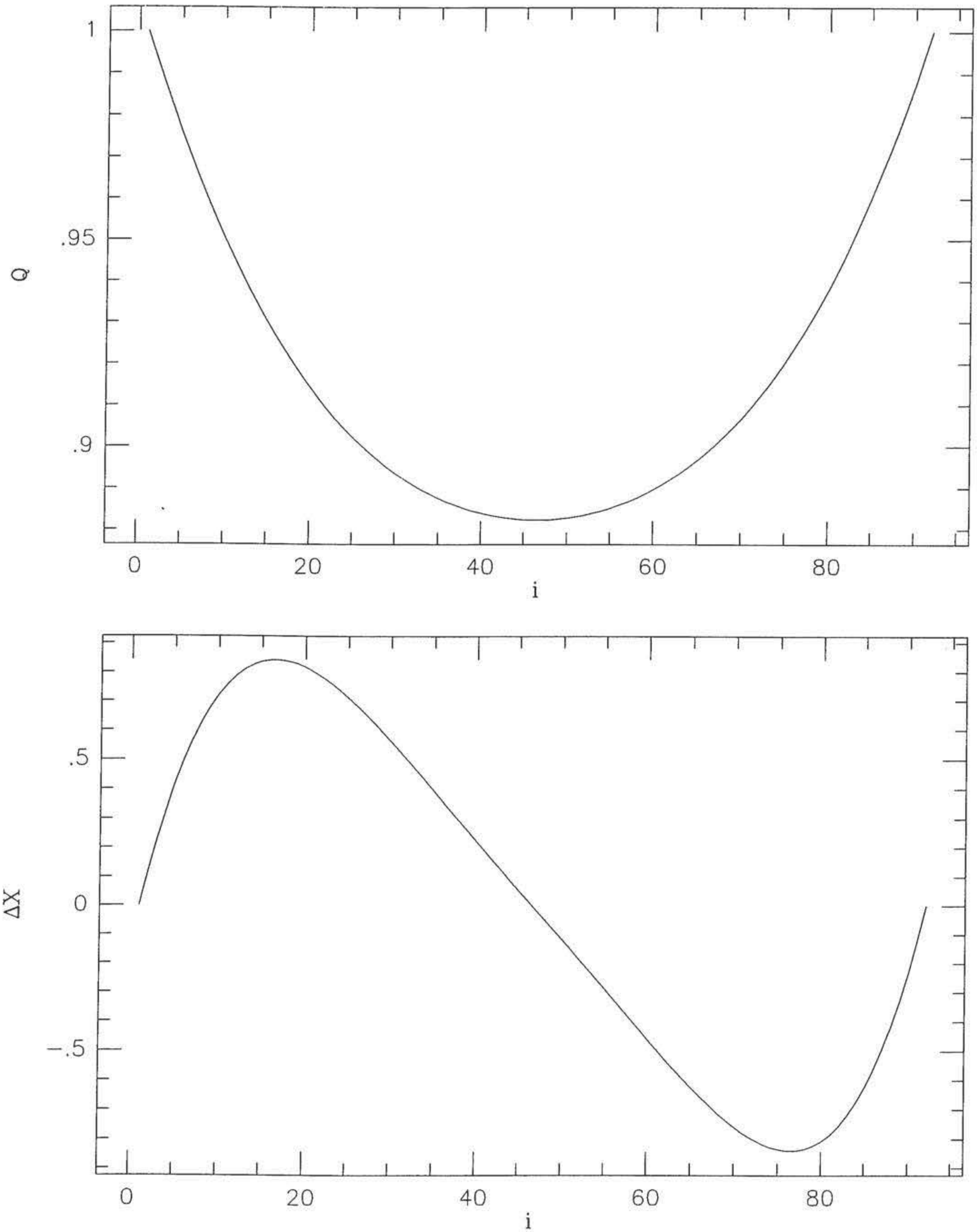
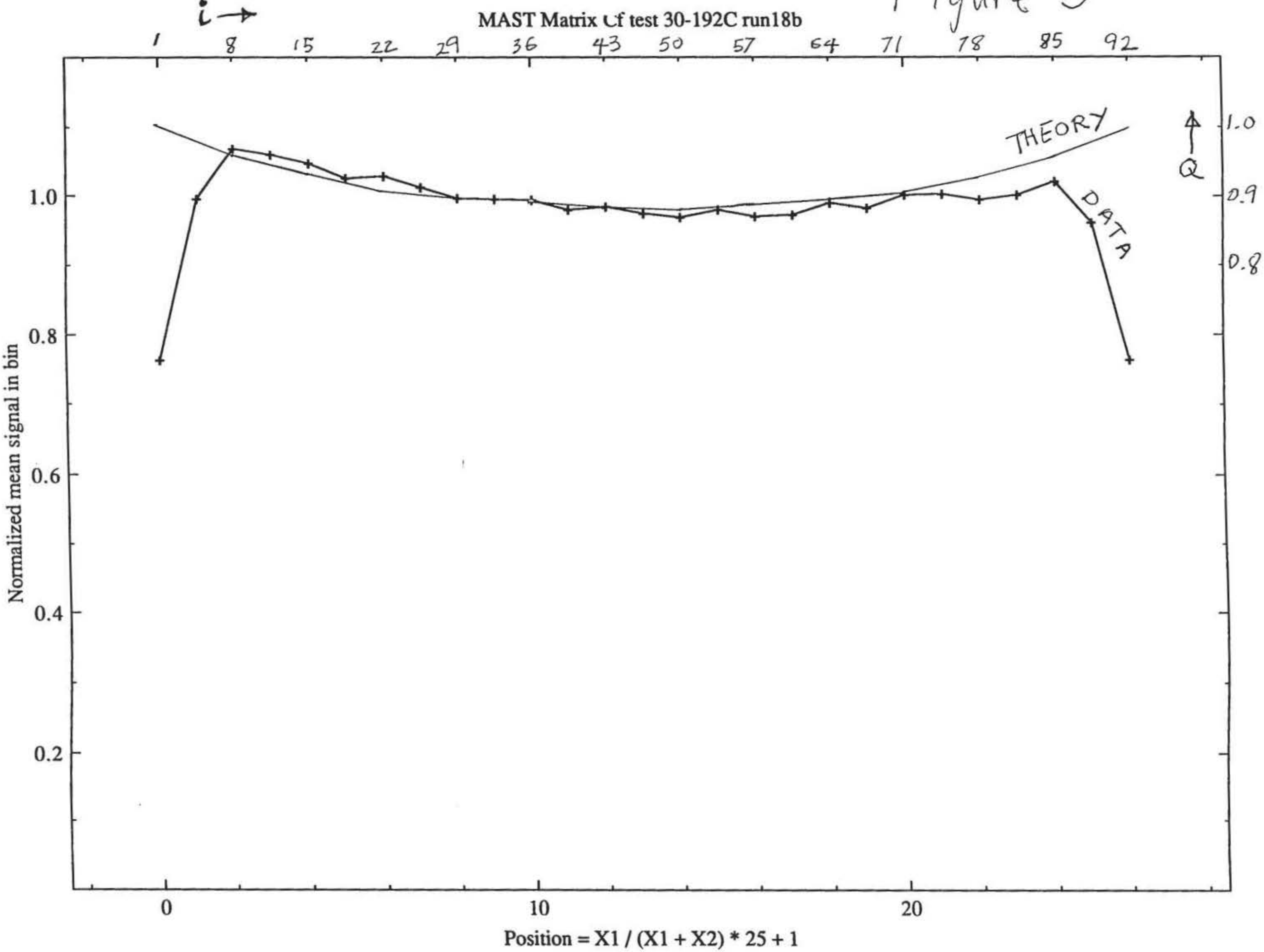


Figure 5


MAST Matrix Cf test 30-192C run18b
i
1
8
15
22
29
36
43
50
57
64
71
78
85
92
THEORY
DATA
Q
1.0
0.9
0.8
Normalized mean signal in bin
0.2
0.4
0.6
0.8
1.0
0
10
20
Position = X1 / (X1 + X2) * 25 + 1

i	Q	X	Xtrue	DX
1	1.000000	1.000000	1.000000	0.000000
2	0.993696	0.990318	0.989011	0.120234
3	0.987598	0.980519	0.978022	0.229682
4	0.981713	0.970603	0.967033	0.328414
5	0.976036	0.960574	0.956044	0.416800
6	0.970568	0.950442	0.945055	0.495572
7	0.965309	0.940205	0.934066	0.564775
8	0.960253	0.929873	0.923077	0.625232
9	0.955398	0.919448	0.912088	0.677140
10	0.950742	0.908936	0.901099	0.721004
11	0.946282	0.898342	0.890110	0.757360
12	0.942016	0.887669	0.879121	0.786462
13	0.937938	0.876925	0.868132	0.808950
14	0.934047	0.866111	0.857143	0.825078
15	0.930338	0.855234	0.846154	0.835370
16	0.926804	0.844300	0.835165	0.840393
17	0.923442	0.833311	0.824176	0.840432
18	0.920251	0.822274	0.813187	0.835984
19	0.917227	0.811192	0.802198	0.827457
20	0.914365	0.800070	0.791209	0.815234
21	0.911661	0.788912	0.780220	0.799661
22	0.909107	0.777721	0.769231	0.781104
23	0.906700	0.766501	0.758242	0.759888
24	0.904437	0.755256	0.747253	0.736331
25	0.902313	0.743988	0.736264	0.710673
26	0.900318	0.732702	0.725275	0.683265
27	0.898452	0.721397	0.714286	0.654202
28	0.896708	0.710079	0.703297	0.623944
29	0.895086	0.698751	0.692308	0.592808
30	0.893577	0.687414	0.681319	0.560761
31	0.892180	0.676068	0.670330	0.527887
32	0.890889	0.664716	0.659341	0.494568
33	0.889701	0.653361	0.648352	0.460844
34	0.888614	0.642003	0.637363	0.426911
35	0.887621	0.630641	0.626374	0.392639
36	0.886723	0.619279	0.615385	0.358251
37	0.885916	0.607914	0.604396	0.323720
38	0.885199	0.596550	0.593407	0.289179
39	0.884568	0.585186	0.582418	0.254714
40	0.884021	0.573823	0.571429	0.220310
41	0.883554	0.562463	0.560440	0.186180
42	0.883166	0.551104	0.549451	0.152088
43	0.882859	0.539746	0.538462	0.118139
44	0.882629	0.528389	0.527473	8.42834
45	0.882475	0.517033	0.516484	5.05481
46	0.882400	0.505678	0.505495	1.68512E-02
47	0.882400	0.494322	0.494505	-1.68539E-02
48	0.882475	0.482967	0.483516	-5.05508E-02
49	0.882629	0.471611	0.472527	-8.42861E-02
50	0.882859	0.460254	0.461538	-0.118147
51	0.883166	0.448896	0.450549	-0.152086
52	0.883554	0.437537	0.439560	-0.186183
53	0.884021	0.426177	0.428571	-0.220316
54	0.884568	0.414814	0.417582	-0.254720
55	0.885199	0.403450	0.406593	-0.289187
56	0.885916	0.392086	0.395604	-0.323723
57	0.886723	0.380721	0.384615	-0.358251
58	0.887621	0.369359	0.373626	-0.392639
59	0.888614	0.357997	0.362637	-0.426911
60	0.889701	0.346639	0.351648	-0.460847
61	0.890889	0.335284	0.340659	-0.494566
62	0.892180	0.323932	0.329670	-0.527887
63	0.893577	0.312586	0.318681	-0.560767
64	0.895086	0.301249	0.307692	-0.592810

65	0.896708	0.289921	0.296703	-0.623949
66	0.898452	0.278603	0.285714	-0.654208
67	0.900318	0.267298	0.274725	-0.683265
68	0.902313	0.256012	0.263736	-0.710675
69	0.904437	0.244744	0.252747	-0.736331
70	0.906700	0.233499	0.241758	-0.759884
71	0.909107	0.222279	0.230769	-0.781113
72	0.911661	0.211088	0.219780	-0.799660
73	0.914365	0.199930	0.208791	-0.815241
74	0.917227	0.188808	0.197802	-0.827456
75	0.920251	0.177726	0.186813	-0.835987
76	0.923442	0.166689	0.175824	-0.840429
77	0.926804	0.155700	0.164835	-0.840399
78	0.930338	0.144766	0.153846	-0.835372
79	0.934047	0.133889	0.142857	-0.825079
80	0.937938	0.123075	0.131868	-0.808950
81	0.942016	0.112331	0.120879	-0.786465
82	0.946282	1.0165E-01	0.109890	-0.757362
83	0.950742	9.1064E-02	9.8901E-02	-0.721003
84	0.955398	8.0551E-02	8.7912E-02	-0.677141
85	0.960253	7.0127E-02	7.6923E-02	-0.625234
86	0.965309	5.9795E-02	6.5934E-02	-0.564776
87	0.970568	4.9558E-02	5.4945E-02	-0.495572
88	0.976036	3.9425E-02	4.3956E-02	-0.416801
89	0.981713	2.9397E-02	3.2967E-02	-0.328414
90	0.987598	1.9481E-02	2.1978E-02	-0.229682
91	0.993696	9.6820E-03	1.0989E-02	-0.120238
92	1.00000	0.000000	0.000000	0.00000