Internal Report
#108

SRL Internal Memo

Electron Capture Half-life Corrections

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A calculation of the transport of cosmic rays through the Galaxy requires attention to the decay of radioactive isotopes. Of these, the pure electron capture decays are particularly interesting because they can only occur before the particles are accelerated and fully stripped, or if they re-attach an electron in flight.

Electron capture half-lives are listed in the Table of Isotopes for laboratory measurements on atoms with all their electrons attatched. Decay can occur via capture of either innermost K-shell electron, or from capture of an L or higher shell electron. This is very different from the case in cosmic rays where the decaying atom will likely have only one attatched electron. The listed half-lives must be corrected for the lack of additional electrons if they are to be used in a propagation calculation.

The correction is based on the following argument: Nuclear decay is governed by a transition rate λ (probability/time) that is the sum of the transition probabilities for each of the possible decay channels. In the case of the atom with filled shells,

$$\lambda_{filled} = \lambda_{K_1} + \lambda_{K_2} + \lambda_{L_1} + \lambda_{L_2} + \lambda_{L_3} + ... \tag{1}$$

There is an equal probability of capturing either of the innermost K-shell electrons, $\lambda_{K_1} = \lambda_{K_2} \equiv \lambda_K$. Also, for all isotopes of interest, capture from the L_1 shell is dominant over capture from all higher shells, $\lambda_{L_1} \equiv \lambda_L$. Neglecting these smaller contributions, the capture probability can be written:

$$\lambda_{filled} = 2\lambda_K + \lambda_L \tag{2}$$

In the cosmic rays, there is only one attatched electron, so only one decay channel: $\lambda_{CR} = \lambda_K$.
The half-life is defined such that $0.5 = exp(-\lambda T_{1/2})$ or $T_{1/2} = (1/\lambda)ln2$.

$$\frac{T_{1/2\ CR}}{T_{1/2\ filled}} = \frac{\lambda_{filled}}{\lambda_{CR}} = 2 + \frac{\lambda_L}{\lambda_K} \tag{3}$$

The last term is simply the L_1/K capture ratio described in Appendix V of the Table of Isotopes (7th Ed., 1978). In this table, the capture ratio is

$$\frac{EC(L_1)}{EC(K)} = k_1(Z)\left[\frac{E(EC) - BE(L_1)}{E(EC) - BE(K)}\right]^2 \tag{4}$$

where E(EC) is the electron capture decay energy and BE(L_1) and BE(K) are the binding energies of the L_1 and K shells respectively. Since in all isotopes of interest the binding energies are small compared to the decay energy, the term in brackets was dropped. The value of $k_1(Z)$ is fit from the graph in the above reference and is taken to be:

$$k_1(Z) = 9.5 \times 10^{-4} Z + 6.4 \times 10^{-2} \tag{5}$$

All relevant radioactive decays are listed in the following table. Only the pure electron capture modes are affected by this correction. The mode column identifies the type of decay; 1 is a β- decay, 2 is β+, and 3 is pure electron capture. The previous corrected half-lives come from the fortran version of the propagation code that had been used previously.

Z	A (a.m.u.)	Mode	T1/2 Lab (yrs.)	E(EC) DecayE (MeV)	BE(K) K binding (keV)	BE(L1) L1 binding (keV)	$T_{1/2}$ CR's prev. (yrs.)	$T_{1/2}$ CR's new (yrs.)
40	93	1	1.53e6					
40	88	3	0.283	0.670	17.996	2.531		0.594
38	85	3	0.1775	1.065	16.104	2.216		0.372
37	83	3	0.236	0.910	15.200	2.065		0.495
38	82	3	6.995e-2	0.180	16.104	2.216		0.147
36	81	3	2.29e5	0.281	14.323	1.921		5.03e5
34	79	1	6.5e5					
36	76	3	1.69e-3	1.311	14.323	1.921		3.54e-3
34	75	3	0.32794	0.864	12.657	1.652		0.687
33	73	3	0.220	0.341	11.866	1.527		0.461
34	72	3	2.3e-2	0.335	12.657	1.652		4.8e-2
32	71	3	3.129e-2	0.229	11.103	1.414		6.55e-2
32	68	3	0.74146	0.106	11.103	1.414		1.55
31	67	3	8.9298e-3	1.000	10.367	1.299		1.87e-2
26	60	1	1.5e6				3.e5	
28	59	3	7.5e4	1.072	8.333	1.008	1.e5	
27	57	3	0.74412	0.836	7.709	0.925	1.57	1.55
28	56	3	1.664e-2	2.136	8.331	1.009	3.53e-2	3.48e-2
28	56	2	1.261e3					
26	55	3	2.73	0.231	7.110	0.844	5.71	5.70
25	54	3	0.85454	1.377	6.537	0.769	1.81	1.78
25	54	1	1.5e8				8.0e5	
25	53	3	3.74e6	0.597	6.537	0.769	7.88e6	7.81e6
24	51	3	7.5844e-2	0.753	5.989	0.696	0.16	0.158
23	49	3	0.903	0.602	5.464	0.627	1.91	1.88
22	44	3	49.	0.268	4.966	0.560	99.4	102.
20	41	3	1.03e5	0.421	4.038	0.438	2.1e5	2.15e5
18	37	3	9.593e-2	0.813	3.205	0.326	0.203	0.200
17	36	1	3.01e5				3.06e5	
13	26	3	7.4e5	4.004	1.559	0.118	8.46e6	1.54e6
13	26	2	9.0e5				8.72e5	
6	14	1	5730.				5730.	
4	10	1	1.51e6				1.6e6	
4	7	3	0.1459	0.862	0.111	—	0.309	0.302

Fig. 6. Subshell ratios for electron capture

Capture of electrons can occur from any atomic shell (X) for which E(EC), the total energy of the capture transition, is greater than BE(X), the binding energy of the X shell in the daughter nucleus. Theoretical values for the relative intensities of K, L_1, L_2, L_3, and M+N+... capture are given in the formulas and graph below. They are derived from the calculations of Brysk and Rose.[1,2]

L_1/K capture ratio:

For allowed and first-forbidden nonunique transitions

$$EC(L_1)/EC(K) = k_1(Z)[E(EC) - BE(L_1)]^2/[E(EC) - BE(K)]^2,$$

where $k_1(Z)$ is given by Fig. 6. For first-forbidden unique transitions

$$EC(L_1)/EC(K) = k_1(Z)[E(EC) - BE(L_1)]^4/[E(EC) - BE(K)]^4.$$

L_2/L_1 capture ratio:

$EC(L_2)/EC(L_1)$ is independent of decay energy if $E(EC) >> BE(L_1) - BE(L_2)$, which is almost always true in cases of interest. The value for allowed and first-forbidden unique transitions is given directly by Fig. 6. For first-forbidden nonunique transitions, the value may be slightly higher, but not by more than about 20%.

L_3/L_1 capture ratio:

L_3 capture is not possible in allowed transitions, and nearly always negligible in first-forbidden nonunique transitions. For first-forbidden unique transitions

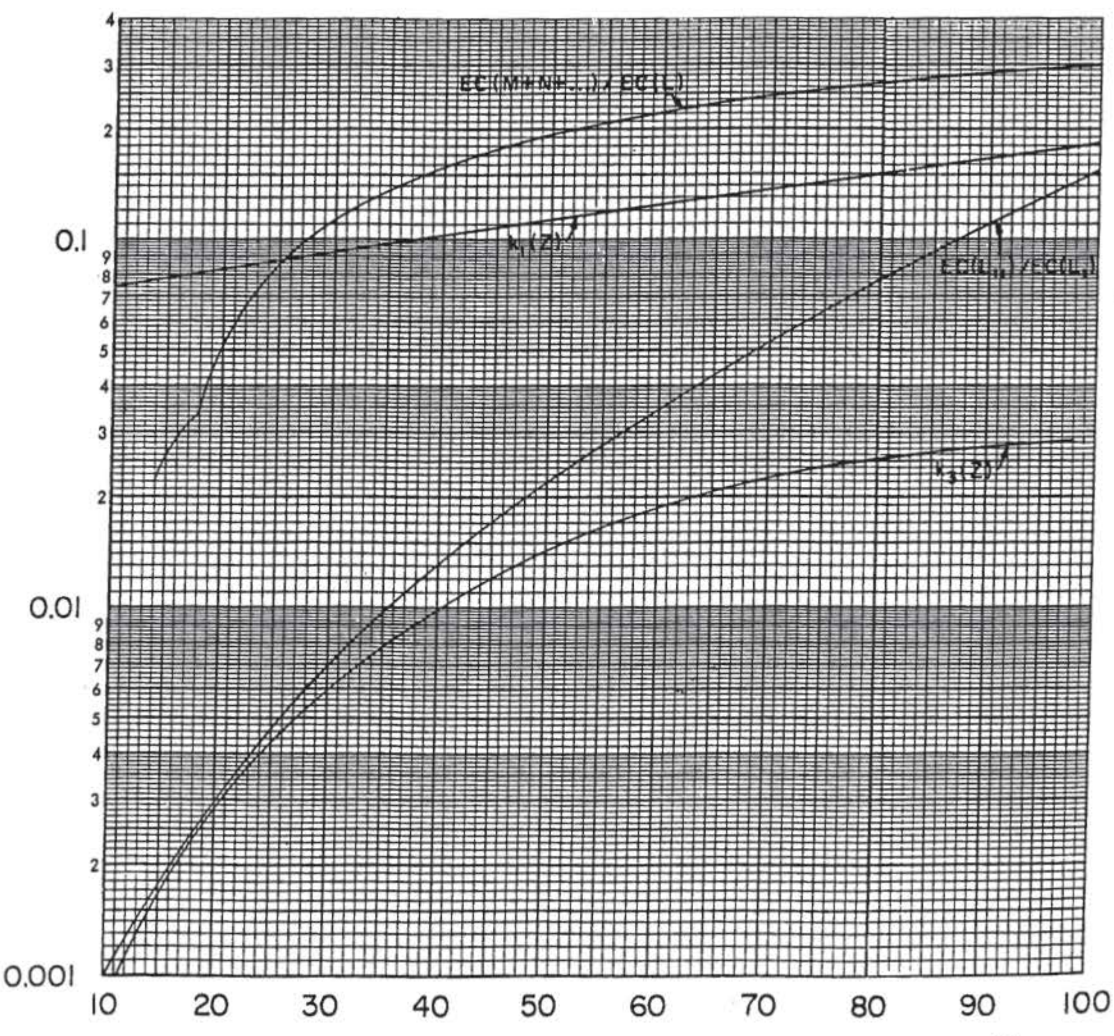
$$EC(L_3)/EC(L_1) = k_3(Z)[E(EC) - BE(L_3)]^2/[E(EC) - BE(L_1)]^4,$$

where $k_3(Z)$ is given by Fig. 6, and E(EC), $BE(L_3)$, and $BE(L_1)$ are in units of MeV.

Capture from higher shells:

The ratio of capture from M and higher shells to capture from the L shell is given directly by Fig. 6 if $E(EC) >> BE(L) - BE(M)$.

1) H. Brysk and M.E. Rose, ORNL-1830(55)
2) A.H. Wapstra, G.J. Nijgh, and R. van Lieshout, *Nuclear Spectroscopy Tables*, North-Holland Publ. Co., Amsterdam (1959)



APPENDIX V