Internal Report #109

SRL Internal Memo
Ionization Losses in an ionized medium
J. George, February 17, 1999

Propagation calculations which describe the transport of cosmic rays through the Galaxy include energy loss terms due to ionization in the interstellar medium. In the past the ISM was typically taken to consist of only neutral hydrogen and helium. More recent calculations have attempted to include the effect of ionized hydrogen which increases the amount of energy loss. This memo attempts to summarize the reasoning for the increased energy loss and state the currently accepted forms for the ionization terms in ionized media.

A recent reference that is typically cited is that of Soutoul, Ferrando, and Webber [1]. In that article they point out in reference to a calculation by Ginzburg and Syrovatskii [2] that the ionization losses in ionized hydrogen are 3.6 times larger than in neutral hydrogen. This overall factor of 3.6 was inserted in at least one version of the propagation code in use at SRL [3].

1 Ginzburg and Syrovatskii

Ginzburg and Syrovatskii explain how the increase in energy loss in an ionized medium comes about. The general form of the ionization term (ignoring shell corrections and the density effect) is [1]:

$$-\frac{dE}{dt} = \frac{4\pi e^4 Z^2 n}{mv}\left\{ln\left[\frac{2mv^2}{I}\left(\frac{E}{Mc^2}\right)^2\right] - \frac{v^2}{c^2}\right\} \tag{1}$$

Here e, m, are the charge and mass of an electron. The energy of the ionizing particle with mass M, charge Z, and velocity v is E. For non-relativistic particles, $E \sim \frac{Mv^2}{2} \ll (M/m)Mc^2$. In a neutral medium with number density n where the ionization potential I is taken to be 15 eV, the above expression reduces to:

$$-\frac{dE}{dt} = 7.62 \times 10^{-9} Z^2 n \sqrt{\frac{2Mc^2}{E_c}}\left\{11.8 + ln(\frac{E_k}{Mc^2})\right\} \text{eV/sec} \tag{2}$$

In a completely ionized medium the ionization potential I is replaced by the plasma frequency $I = \hbar\omega_p = \hbar\sqrt{(4\pi e^2 n/m)}$.

$$-\frac{dE}{dt} = 7.62 \times 10^{-9} Z^2 n \sqrt{\frac{2Mc^2}{E_k}}\left\{ln(\frac{E_k}{Mc^2}) - \frac{1}{2}ln(n) + 38.7\right\} \text{eV/sec} \tag{3}$$

The ratio of the loss in an ionized medium to the loss in a neutral medium is about 3.5 for an energy of 500 MeV/nuc and 1 cm^{-3} ISM hydrogen density.

2 Mannheim and Schlickeiser

The issue is also discussed in a recent paper by Strong and Moskalenko [4]. They refer to a general formula by Mannheim and Schlickeiser [5]. Note there is a typo in the original Mannheim and Schlickeiser paper which is corrected by Strong and Moskalenko.

[1]The original text shows the factor $\frac{E}{Mc^2}$ as $\frac{E}{m_e c^2}$. I believe this is a typographical error.

$$\frac{dE}{dt}(\beta \geq \beta_0) = \frac{-2\pi r_e^2 c m_e c^2 Z^2}{\beta} \sum_{s=H,He} n_s[B_s + B'(\alpha_f Z/\beta)] \tag{4}$$

where

$$\beta_0 = 1.4e^2/\hbar c = 0.01 \tag{5}$$

$$B_s = \left[ln\left(\frac{2m_e c^2 \beta^2 \gamma^2 Q_{max}}{I_s^2}\right) - 2\beta^2 - 2\frac{C_s}{z_s} - \delta_s\right] \tag{6}$$

$$Q_{max} \sim \frac{2m_e c^2 \beta^2 \gamma^2}{1 + (2\gamma m_e/M)} \tag{7}$$

The ionization potentials $I_H = 19eV$ and $I_{He} = 44eV$ are weighted sums over the oscillator strengths of the electron levels in hydrogen and helium. For an ionized medium, again replace these potentials with the plasma frequency. The resulting form for cosmic ray particle energies where $\gamma^2 \sim 1$ is essentially identical to that of Ginzburg and Syrovatskii and also yields a ratio of about 3.5 for the losses in ionized media compared to neutral ones.

3 Nishimura

Yet another reference discussing the ionization issue can be found in Nishimura [6]. Nishimura gives the following formulae:

$$-\frac{dE}{dx} = 4\pi\frac{NZ_0}{A_0}\frac{Z^2e^4}{mv^2}\left[ln\left(\frac{2m\beta^2}{(1-\beta^2)I}\right) - 2\beta^2 - \delta\right] \text{g/cm}^2 \tag{8}$$

in a neutral medium and

$$-\frac{dE}{dx} = 4\pi\frac{NZ_0}{A_0}\frac{Z^2e^4}{mv^2}\left[ln\left(\frac{2m\beta^2}{\sqrt{1-\beta^2}\hbar\omega_p}\right) - \beta^2 + 1\right] \text{g/cm}^2 \tag{9}$$

in an ionized one. Here $\frac{NZ_0}{A_0} = n_s$ is the density of the medium and the plasma frequency is $\omega_p = \sqrt{4\pi n_e e^2/m}$. Note that the Nishimura form is given in units of column density so it contains an extra factor of β^{-1}. This comes in changing from time units to distance units $dx = vdt$. This form is equivalent to the Mannheim and Schlickeiser form except for the factor of γ in the ln term and the coefficient of β^2 in the ionized medium expression. No explanation for how this was derived is given in the reference. For non-relativistic particles it will make little difference.

4 Conclusion

For non-relativistic particles, these versions of the expression for ionization losses are essentially equivalent. Whichever form is calculationally convenient can be used. Of the three, the Mannheim and Schlickeiser form has the best documentation in the reference. Given that we also probably have fairly good access to them for questions, this would be a good choice to use in future propagation code.

References

[1] Soutoul, Ferrando, and Webber. Proc. 21st ICRC, v.3, 337.

[2] Ginzburg and Syrovatskii. in "The Origin of Cosmic Rays," p. 122. (1964).

[3] N. Yanasak. fortran propagation code: prop.ny.f

[4] Strong, A. and I. Moskalenko. ApJ 509: 212-228 (1998)

[5] Mannheim, K. and R. Schlickeiser. A&A 286:983-996 (1994).

[6] Nishimura, J. in "Cosmic Ray Astrophysics," p. 99. M. Oda, J. Nishimura, and K. Sakurai, eds. Terra Scientific, Tokyo (1988).