

B-L PROGRAM WITH EXTERNAL SOURCES

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This report describes modifications made to the B-L program (INVAR) to include the effects of external sources in tracing field lines and calculating real-field B and L values. The external sources presently being considered are: 1) the magnetopause current (Mead, 1964), and 2) the neutral sheet current (Williams & Mead, 1965).

The basic line-tracing program LOCAL, and the associated routines START and LINES, were modified to provide for additional field sources. The routine INTEG, which calculates the integral invariant I, and the routine CARMEL, which calculates L, are unchanged. The magnetic field components are calculated by several routines: 1) BGSFC/FIELD calculates the field due to internal sources, and returns the components in geocentric coordinates; 2) BMEAD calculates the field due to the magnetopause current, and returns the components in dipole coordinates; 3) BCS calculates the field due to the neutral sheet current, and returns the components in dipole coordinates; 4) XFORM transforms field components from dipole to geocentric coordinates. Other required routines are METCON, which calculates MLT, and RDICON/DICON, which performs coordinate transformations. Required input parameters are:

- 1) Parameters specifying the time for which the field is to be evaluated:

YEAR, TM: e.g. 1967.3,

DAY: e.g. 211.,

UTSEC: universal time (seconds), e.g. 36400.3;

2) Parameters specifying the external field sources:

RB: stand-off distance of sub-solar point of magnetopause,
e.g. 10.6 (earth radii),

RIN: distance to inner edge of neutral sheet, e.g. 10.0
(earth radii),

ROUT: distance to outer edge of neutral sheet, e.g. 40.0
(earth radii),

BSHEET: value of B near center of neutral sheet, in gammas,
e.g. 40.0,

CUTOFF^{*}: the approximate thickness of the current sheet,
e.g. 0.01;

3) Parameters specifying the location in space at which
the field is to be evaluated:

ALT: geodetic altitude in kilometers,

THIN: geocentric colatitude in degrees,

PHIN: geocentric longitude in degrees;

4) Parameters controlling the evaluation:

ERR: controls the step size used in integrating the
differential equations for the field line;
normally set to 0.005 or 0.01,

NMX: the maximum order spherical harmonics to be used to
evaluate the internal field, normally set to 11,

KPRINT: line trace is printed if KPRINT \neq 0;

5) Parameters which are not normally varied:

$A1^*$: the geocentric longitude of the north magnetic pole, in degrees,

$A2^*$: the geocentric colatitude of the north magnetic pole, in degrees.

$G10$ thru $G65^*$: coefficients for the field due to the magnetopause current (Schmidt normalized).

(* - Routines containing these parameters must be recompiled if the values are to ^{be} changed.)

These parameters (except those marked with a *) are contained either in COMMON/VARCOM/... or in the calling sequence of LOCAL. All parameters except RB (and *) can be changed at any time. The value of RB at the beginning of the program will be used throughout.

SUBROUTINE BMEAD

BMEAD calculates the field due to the magnetopause current by the method given by Mead (Mead, 1964). With a few slight changes in notation, Mead's equations (3) thru (6) can be written:

$$(3') \quad T_N = \sum_{M=0}^N (G_{NM} \cos M\theta + H_{NM} \sin M\theta) P_{NM}(\cos \theta),$$

$$(4'-6') \quad P_{NM}(\mu) = K_{NM} \frac{1}{2^N N!} (1-\mu^2)^{M/2} \frac{d^{N+M}}{d\mu^{N+M}} (\mu^2 - 1)^N,$$

$$K_{NM} = \begin{cases} \left(\frac{2(N-M)!}{(N+M)!} \right)^{1/2} & \text{if } m > 0 \\ 1 & \text{if } m = 0 \end{cases},$$

where θ is the dipole colatitude, $\mu = \cos \theta$, and $\emptyset = (\pi/12)$ MLT.

The magnetic potential is then

$$(1') \quad V_{\text{MEAD}} \approx a \sum_{N=1}^6 (r/a)^N T_N(\theta, \emptyset),$$

where a is the radius of the earth. B is then the negative gradient of V , with the following components referring to a dipole coordinate system, with \emptyset measured from local midnight:

$$(7') \quad B_R = - \sum_{N=1}^6 R^{N-1} \sum_{M=0}^N G_{NM} \cos(M\emptyset) P_{NM}(\mu),$$

$$(8') \quad B_\theta = \sum_{N=1}^6 R^{N-1} \sum_{M=0}^N G_{NM} \cos(M\emptyset) \sin \theta \frac{d}{d\mu} P_{NM}(\mu),$$

$$(9') \quad B_\emptyset = \frac{1}{\sin \theta} \sum_{N=1}^6 R^{N-1} \sum_{M=0}^N G_{NM} M \sin(M\emptyset) P_{NM}(\mu),$$

where $R = r/a$, and the H_{NM} have been suppressed since they are all zero. The expressions used for P_{NM} are listed in table 1.

SUBROUTINE BCS

BCS calculates the field due to the current flowing in a sheet in the (magnetic) equatorial plane. The sheet is assumed to contain a uniform current flowing in a direction normal to the earth-sun line. See fig. 5 of Williams & Mead (Williams & Mead, 1965) for a detailed description of the coordinates used. (cf. Appendix I, this report).

SUBROUTINE XFORM

XFORM transforms the r, θ, ϕ components of a vector, expressed in dipole coordinates, to r, θ, ϕ components expressed in geocentric coordinates. The transformation is done in four steps:

- 1) Subroutine RDICON is used to convert dipole coordinates (θ_D, ϕ_D) to geocentric coordinates (θ_G, ϕ_G) .
- 2) Dipole spherical components are transformed to dipole cartesian components.
- 3) Dipole cartesian components are transformed to geocentric cartesian components.
- 4) Geocentric cartesian components are transformed to geocentric spherical components.

The relations used are (with V any arbitrary vector):

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{DIPOLE}} = \begin{pmatrix} \sin \theta_D \cos \phi_D & \cos \theta_D \cos \phi_D & -\sin \phi_D \\ \sin \theta_D \sin \phi_D & \cos \theta_D \sin \phi_D & \cos \phi_D \\ \cos \theta_D & -\sin \theta_D & 0 \end{pmatrix} \begin{pmatrix} V_r \\ V_\theta \\ V_\phi \end{pmatrix}_{\text{DIPOLE}} \quad (2)$$

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEO.}} = \begin{pmatrix} \cos A2 \cos A1 & -\sin A1 & \sin A2 \cos A1 \\ \cos A2 \sin A1 & \cos A1 & \sin A2 \sin A1 \\ -\sin A2 & 0 & \cos A2 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{DIPOLE}} \quad (3)$$

$$\begin{pmatrix} V_r \\ V_\theta \\ V_\phi \end{pmatrix}_{\text{GEO.}} = \begin{pmatrix} \sin \theta_G \cos \phi_G & \sin \theta_G \sin \phi_G & \cos \theta_G \\ \cos \theta_G \cos \phi_G & \cos \theta_G \sin \phi_G & -\sin \theta_G \\ -\sin \phi_G & \cos \phi_G & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEO.}} \quad (4)$$

The conversion (1) is done by a method analogous to transforming the position vector (X,Y,Z) using relation (3). $V_{r,\text{GEO.}}$ is compared to $V_{r,\text{DIPOLE}}$ to see that they are equal. The transformation loses accuracy at certain positions, and the routine generates an error message if the error is greater than one part in 10^3 . This happens quite rarely, primarily when $V_r \ll |V|$.

USAGE

To use this program, include the following decks:

LOCAL/M, START/M, LINES/M, INTEG(V.2), CARMEL, BGSFC, FIELD(N<11), BCS, BMEAD, XFORM, DISET, RDICON/DICON/M, and METCON (V.3). Calling sequence:

```
COMMON/VARCOM/YEAR, DAY, UTSEC, RB, RIN, ROUT, BSHEET, KPRINT
COMMON/LOCL/BSAVE(200), POS(6,200)
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⋮

set up values in /VARCOM/, plus ALT, THIN, PHIN, ERR, TM, NMX

⋮

```
CALL LOCAL(ALT, THIN, PHIN, ERR, TM, NMX, KMIN, JMAX, BB, FL)
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RETURNS:

JMAX = number of points generated,

KMIN = subscript for point with minimum B value,

BB = B value at input coordinates (in Gauss),

FL = L value at input coordinates (in earth radii),

BSAVE(J) = B value at point J (in Gauss),

POS(1,J) = radial distance of point J (in earth radii),

POS(2,J) = geocentric colatitude of point J (in radians),
POS(3,J) = geocentric longitude of point J (in radians),
POS(4,J) = dipole latitude of point J (in degrees),
POS(5,J) = dipole longitude of point J (in degrees),
POS(6,J) = MLT of point J (in hours).

Note: If field line is not closed, or ERR is so small that more than 200 points are required, FL will be set to -1.0. If KPPINT \neq 0, an error message will also be printed.

J=2 corresponds to the input point. J=JMAX < 200 corresponds to the first point which has a B value greater than or equal to BSAVE(2).

STORAGE REQUIREMENTS:

LOCAL/M (V.3)	1C78 ₁₆ = 7288	DISET	308 ₁₆ = 776
START/M	AE8 ₁₆ = 2792	RDICON/DICON/M	71A ₁₆ = 1818
LINES/M (V.4)	1000 ₁₆ = 4096	METCON(V.3)	418 ₁₆ = 1048
INTEG(V.2)	73A ₁₆ = 1850	/VARCOM/	20 ₁₆ = 32
CARMEL	62E ₁₆ = 1582	/LOCL/	15E0 ₁₆ = 5600
BGSFC	1166 ₁₆ = 4454	/COORDS/	20 ₁₆ = 32
FIELD(N=11,FAST)	27BE ₁₆ = 10174	/ANGLES/	20 ₁₆ = 32
BCS	5A6 ₁₆ = 1446	/COEFFS/	1E4 ₁₆ = 484
BMEAD	A86 ₁₆ = 2694	/FLDCOM/	28 ₁₆ = 40
XFORM	354 ₁₆ = 852		

TOTAL 47,090₁₀ plus system routines

TIMING: 300-800 milliseconds, roughly proportional to L.

Listings and decks are available in 211 Downs.

Note: See writeup for METD/METG (V.5) for a fast approximation to get similar results.

BIBLIOGRAPHY:

1. Brown, "Subroutine METG/METD (V.5)," Internal, SRL, 1970.
2. Mead, JGR, 69, 1182 (1964).
3. Williams & Mead, JGR, 70, 3024 (1965).

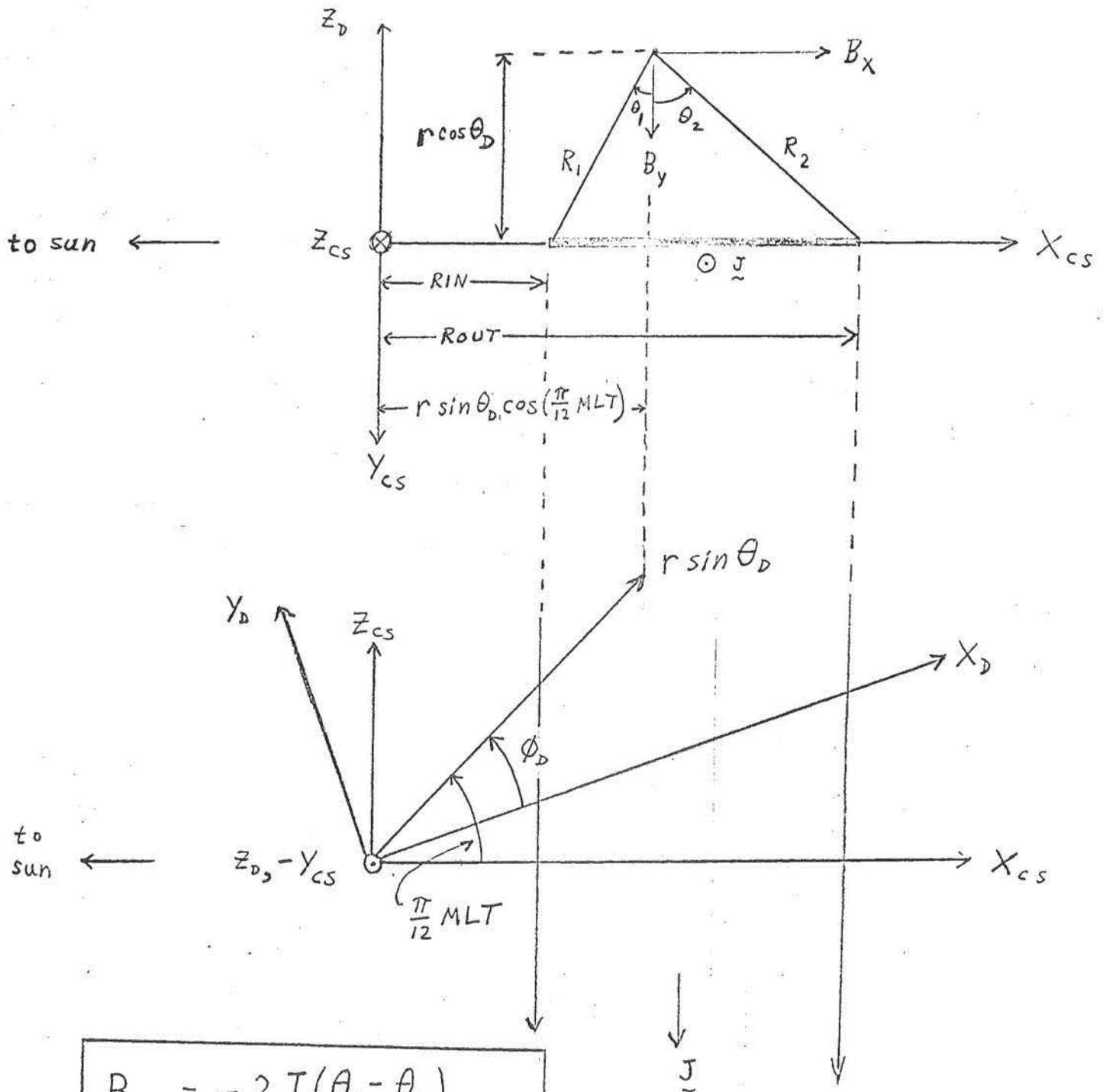
Table 1

Schmidt Normalized Associated Legendre Polynomials

<u>N</u>	<u>M</u>	<u>$P_N^M(x)$</u>
1	0	x
2	1	$1.732051x(1-x^2)^{1/2}$
3	0	$2.5x^3 - 1.5x$
3	2	$1.936491x(1-x^2)$
4	1	$(5.533985x^3 - 2.371708x)(1-x^2)^{1/2}$
4	3	$2.09165x(1-x^2)^{3/2}$
5	0	$7.875x^5 - 8.75x^3 + 1.875x$
5	2	$(7.68522x^3 - 2.56174x)(1-x^2)$
5	4	$1.568738x(1-x^2)^2$
6	1	$(18.90312x^5 - 17.18466x^3 + 2.86411x)(1-x^2)^{1/2}$
6	3	$(9.96282x^3 - 2.717133x)(1-x^2)^{3/2}$
6	5	$2.326814x(1-x^2)^{5/2}$

APPENDIX I

Current Sheet Coordinates



$$B_{X_{CS}} = -2J(\theta_2 - \theta_1)$$

$$B_{Y_{CS}} = 2J \ln(R_2/R_1)$$