PATH LENGTH DISTRIBUTION AND GEOMETRICAL FACTOR AS FUNCTION OF POLAR ANGLE

by Steward Hartman

Internal Report No. 2
May 1974
(Supersedes Internal Report No. 2 of 8/68)

I. Geometrical factor as a function of the polar angle of the trajectory:

A. For a single disk:

$$\frac{d}{d\theta}(A\Omega) = 2\pi \sin\theta \cos\theta \text{ (Area)}.$$

Obviously $(A\Omega) = \int_0^{\pi/2} \frac{d}{d\theta}(A\Omega) = \pi(\text{Area})$

B. For two disks on the same axis.

Here we must take account of the fact that at a given polar angle $\theta$ not all of the elements of area of the top disk will have corresponding elements of area on the bottom disk.

Let r describe the intersection point on the top disk and $(\theta,\phi)$ the trajectory angles. Then as the azimuthal angle $\phi$ is rotated over $2\pi$, not all values of $\phi$ will give a trajectory that intersects the bottom disk. The geometry for the two disks is shown in Figure 1.

We have the following four cases for the two disks.

Let $T = \tan\theta$; x,y,L as in Figure 1,

1. If $T \geq x + y$; $\frac{d(A\Omega)}{d\theta} = 0$.

2.a If $y > x$, and $T \leq y - x$;

$$\frac{d(A\Omega)}{d\theta} = 2\pi^2 x^2 L^2 \sin\theta \cos\theta.$$

2.b If $x > y$, and $T \leq x - y$;

$$\frac{d(A\Omega)}{d\theta} = 2\pi^2 y^2 L^2 \sin\theta \cos\theta.$$

3. If $T < x + y$,

and $T > y - x$,

and $T > x - y$;

$$\frac{d(A\Omega)}{d\theta} = 2\pi L^2 \sin\theta \cos\theta \, F(x,y,T),$$

where

$$F(x,y,T) = x^2 \tan^{-1}\left[\frac{\left\{4x^2y^2 - (x^2+y^2-T^2)^2\right\}^{1/2}}{x^2-y^2+T^2}\right]$$

$$- \frac{1}{2}\left\{4x^2y^2 - (x^2+y^2-T^2)^2\right\}^{1/2}$$

$$+ y^2 \tan^{-1}\left[\frac{\left\{4x^2y^2 - (x^2+y^2-T^2)^2\right\}^{1/2}}{y^2-x^2+T^2}\right].$$

For $x = y$, only case 1 or 3 applies. Case 3 then reduces to

$$F(x,T) = 2x^2 \tan^{-1}\left[\left(\frac{4x^2}{T^2} - 1\right)^{1/2}\right]$$

$$- \frac{T^2}{2}\left(\frac{4x^2}{T^2} - 1\right)^{1/2}.$$

II. Path length distribution for one or two disks.

We are given a disk of a certain thickness s and the differential geometrical factor $\frac{d(A\Omega)}{d\theta}$ .

We wish to find $\frac{dN(\ell)}{d\ell}$ the number of particles of path lengths between $\ell$ and $\ell + d\ell$.

For an isotropic incident flux

$$\frac{dN}{d\ell} = (\text{Flux})\ (\text{Time})\ \frac{d(A\Omega)}{d\theta}\ \frac{d\theta}{d\ell}$$

Since $\ell = s \sec\theta$, we have

$$\frac{d\ell}{d\theta} = s\ \frac{d}{d\theta}(\sec\theta) = s\ \frac{\sin\theta}{\cos^2\theta}\ .$$

then

$$\frac{dN}{d\ell}(\theta) = (\text{Flux})(\text{Time})\ \frac{d(A\Omega)}{d\theta}\ \frac{\cos^2\theta}{\sin\theta}\ \frac{1}{s}\ .$$

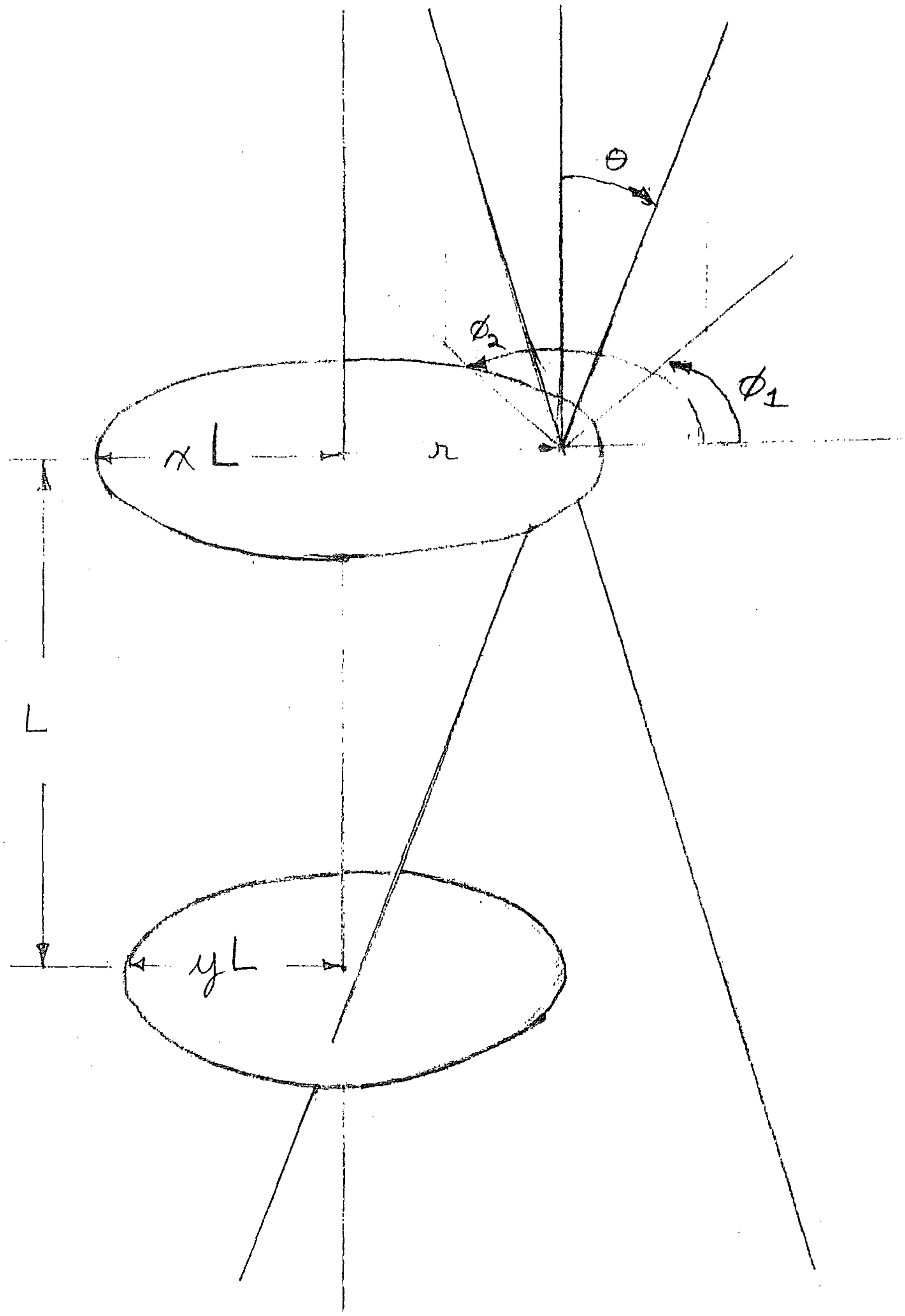


FIGURE 1
DISK CONFIGURATION
FOR $\phi_1$ GET AN INTERSECTION.
FOR $\phi_2$ THE TRAJECTORY MISSES.