I. Geometrical factor as a function of the polar angle of the trajectory:

A. For a single disk:

\[ \frac{d}{d\theta} (A\Omega) = 2\pi \sin \theta \cos \theta \text{(Area)}. \]

\[ \int_0^{\pi/2} \frac{d}{d\theta} (A\Omega) = \pi \text{(Area)} \]

B. For two disks on the same axis.

Here we must take account of the fact that at a given polar angle \( \theta \) not all of the elements of area of the top disk will have corresponding elements of area on the bottom disk.

Let \( r \) describe the intersection point on the top disk and \( (\theta, \phi) \) the trajectory angles. Then as the azimuthal angle \( \phi \) is rotated over \( 2\pi \), not all values of \( \phi \) will give a trajectory that intersects the bottom disk. The geometry for the two disks is shown in Figure 1.

We have the following four cases for the two disks.

Let \( T = \tan \theta; x, y, L \) as in Figure 1,

1. If \( T \geq x + y \); \( \frac{d(A\Omega)}{d\theta} = 0 \).

2.a If \( y > x \), and \( T \leq y - x \);

\[ \frac{d(A\Omega)}{d\theta} = 2\pi x^2 L^2 \sin \theta \cos \theta. \]
2.b If \( x > y \), and \( T \leq x - y \);

\[
\frac{d(\Delta\Omega)}{d\theta} = 2\pi y_1 x_1^2 \sin\theta \cos\theta.
\]

3. If \( T < x + y \),
and \( T > y - x \),
and \( T > x - y \);

\[
\frac{d(\Delta\Omega)}{d\theta} = 2\pi L^2 \sin\theta \cos\theta F(x,y,T),
\]

where

\[
F(x,y,T) = x^2 \tan^{-1} \left[ \frac{\left\{ \left( \frac{2y^2 - (2y - T)^2}{x^2 - y^2 + T^2} \right) \right\}^{1/2}}{x^2 - y^2 + T^2} \right] - \frac{1}{2} \left\{ \frac{4x^2 - (x^2 + y^2 - T^2)^2}{x^2 - y^2 + T^2} \right\}^{1/2} + y^2 \tan^{-1} \left[ \frac{\left\{ \left( \frac{4x^2 - (x^2 + y^2 - T^2)^2}{y^2 - x^2 + T^2} \right) \right\}^{1/2}}{y^2 - x^2 + T^2} \right].
\]

For \( x = y \), only case 1 or 3 applies. Case 3 then reduces to

\[
F(x,T) = 2x^2 \tan^{-1} \left[ \left( \frac{4x^2}{T^2} - 1 \right)^{1/2} \right] - \frac{T^2}{2} \left( \frac{4x^2}{T^2} - 1 \right)^{1/2}.
\]
II. Path length distribution for one or two disks.

We are given a disk of a certain thickness $s$ and the differential geometrical factor $\frac{d(A\Omega)}{d\theta}$.

We wish to find $\frac{dN(\ell)}{d\ell}$ the number of particles of path lengths between $\ell$ and $\ell + d\ell$.

For an isotropic incident flux

$$\frac{dN}{d\ell} = (\text{Flux})(\text{Time}) \frac{d(A\Omega)}{d\theta} \frac{d\theta}{d\ell}$$

Since $\ell = s \sec \theta$, we have

$$\frac{d\ell}{d\theta} = s \frac{d}{d\theta} (\sec \theta) = s \frac{\sin \theta}{\cos^2 \theta}.$$

Then

$$\frac{dN}{d\ell} (\theta) = (\text{Flux})(\text{Time}) \frac{d(A\Omega)}{d\theta} \frac{\cos^2 \theta}{\sin \theta} \frac{1}{s}.$$
FIGURE 1

DISK CONFIGURATION

FOR $\phi_1$ GET AN INTERSECTION.

FOR $\phi_2$ THE TRAJECTORY MISSES.