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ADIABATIC ENERGY LOSS OF SOLAR-COSMIC-RAY PROTONS

by

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ABSTRACT:

The purpose of this project is to determine the amount of energy lost through adiabatic deceleration by a solar flare particle during propagation from the sun to the earth. It is then possible to estimate the energy the particles observed at earth had at the sun. A numerical solution to the Fokker-Planck propagation equation for steady-state continuous emission is used to investigate energy change. This steady-state solution is related to the time-integrated density observed at earth for impulsively injected flares. It is found that the amount of energy lost can strongly depend on the assumed radial dependence of the diffusion coefficient. Assuming the diffusion coefficient to be independent of radius, particles lose roughly 50% of their energy propagating from sun to earth.

METHOD:

An ideal way to determine how much energy is lost by a solar flare particle during propagation from the sun to the earth would be to inject impulsively mono-energetic particles at the sun and to follow them as they propagate to earth. There are two fundamental problems with this method, however. First, this ideal method involves three independent variables -- time, radius, and energy; this makes numerical calculations time-consuming (expensive). Second, delta functions (in energy here) are difficult to deal with numerically.

These two problems are avoided using the following techniques. The first is circumvented by solving the corresponding time-independent problem with continuous rather than impulsive injection. This reduces the number of independent variable to only two -- radius and energy. It will be shown later how to relate the steady-state density solution at earth to the time-independent problem with continuous rather than impulsive injection.

The second problem is avoided by taking finite energy widths rather than delta functions. This finite energy bin is produced by injecting two different energy spectra at the sun, computing the density at earth using the appropriate propagation equation for each input spectrum, and relating the changes in the computed densities to the changes in the input spectra. A typical pair of spectra is shown on the following page. Since the only difference in the continuous injection rate is that 9-10 MeV particles are injected in Case I and not in Case II, and the problem is linear, one can subtract the density at earth in Case II from the density at earth in Case I to find the density that would be observed for the injection of only 9-10 MeV particles. Thus one can discover at what energy particles of 9-10 MeV at the sun are observed at the earth.

A fundamental part of this procedure is the determination of the density at earth as a function of energy. This is done using the time-independent Fokker-Planck propagation equation with appropriate boundary conditions. The equation solved is:

$$\frac{\partial n}{\partial t} = 0 = \frac{\partial}{\partial r}(r^2 \kappa \frac{\alpha n}{\partial r} - \frac{\partial}{\partial r}(r^2 V n) + \frac{1}{3}\frac{\partial}{\partial r}\left[r^2 V\right]\frac{\partial}{\partial T}(\alpha T n)$$

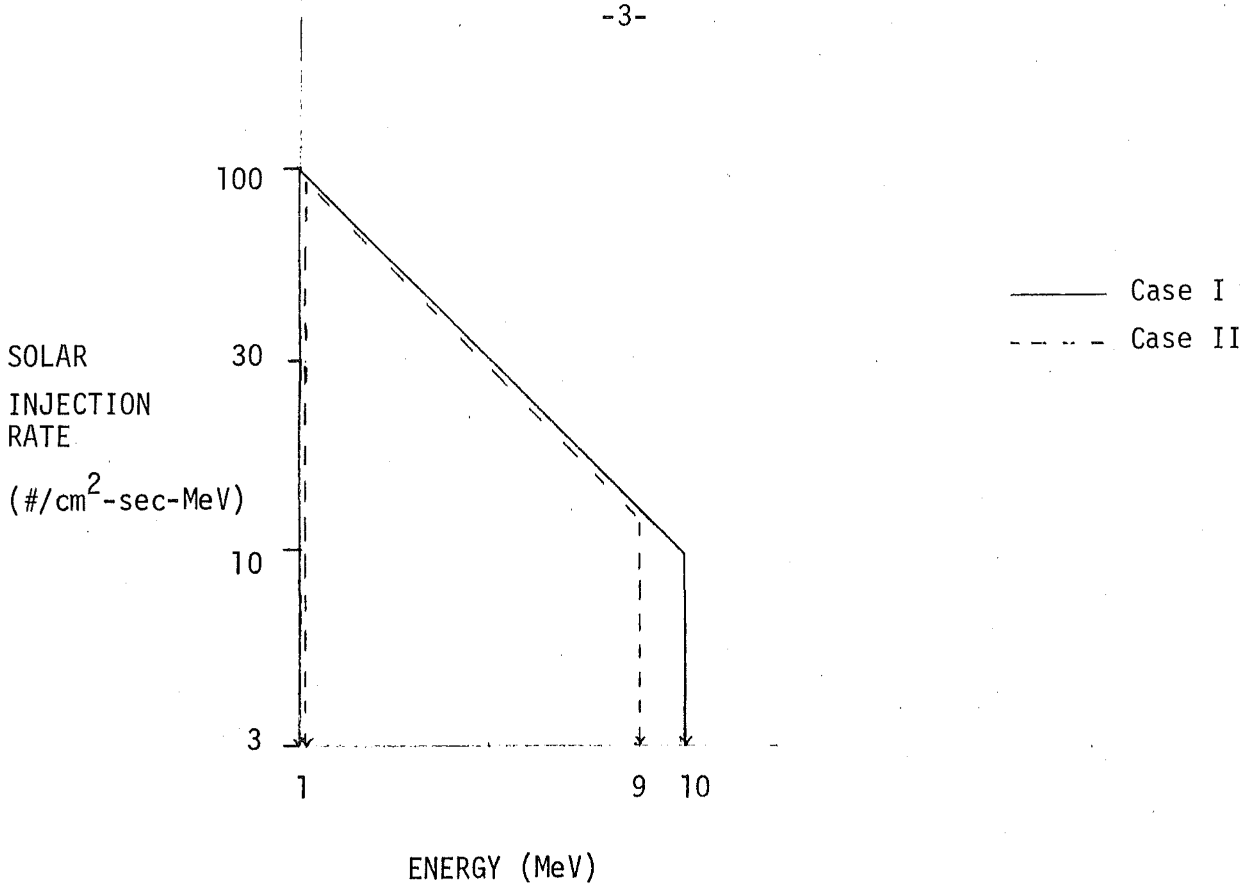
n - particle density

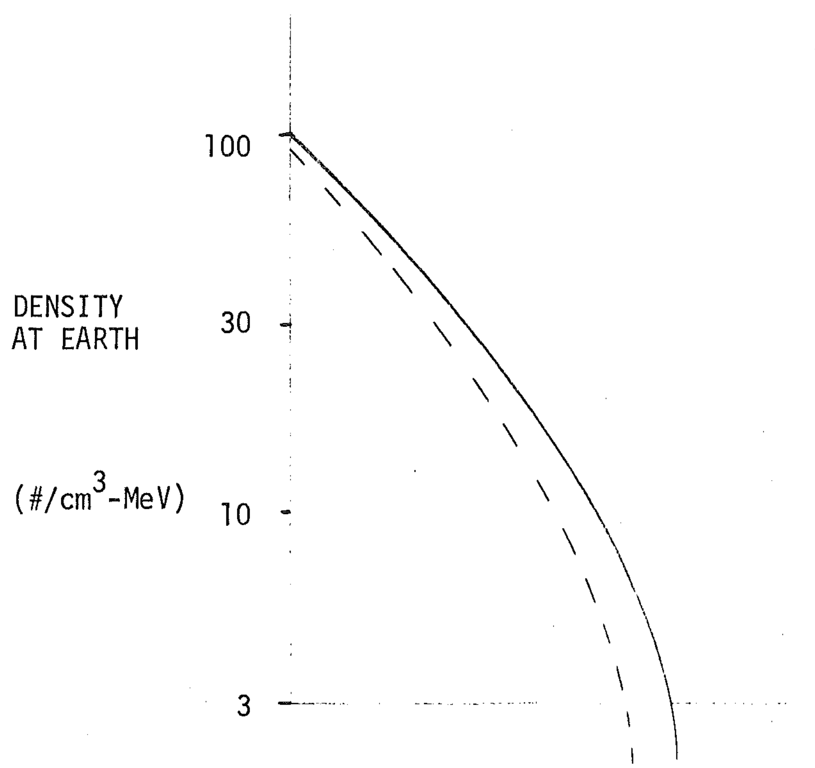
V = solar wind velocity

$\alpha = (T + 2\ mc^2)/(T + mc^2)$

$k = k_{rr}$ = radial diffusion coefficient

Several simplifying assumptions have been made in order to make the problem tractable. There is azimuthal symmetry. The diffusion coefficient is independent of energy. There is a free escape boundary at a radius L where the density of particles is zero for all energies. The solar wind


100
30
10
3
SOLAR
INJECTION
RATE
(#/cm2-sec-MeV)
1
9
10
ENERGY (MeV)
Case I
Case II


100
30
10
3
DENSITY
AT EARTH
(#/cm3-MeV)

velocity is assumed independent of radius. Different radial dependences of solar wind velocity are also easily handled by the formalism. α is set equal to 2.

The particles are injected at $r = 1$ solar radius. The continuous injection rate is determined by setting the flux, F, where

$$F = -k\, dn/dr + CVn \qquad \text{(C is the Compton-Getting factor)}$$

to the desired value. This is thought to be equivalent to a reflecting boundary at the sun with a constant injection of particles into interplanetary space. The flux due to the Compton-Getting effect is not entirely understood, but is usually small. A more realistic model would have the solar wind velocity zero at the sun, thus eliminating the problem.

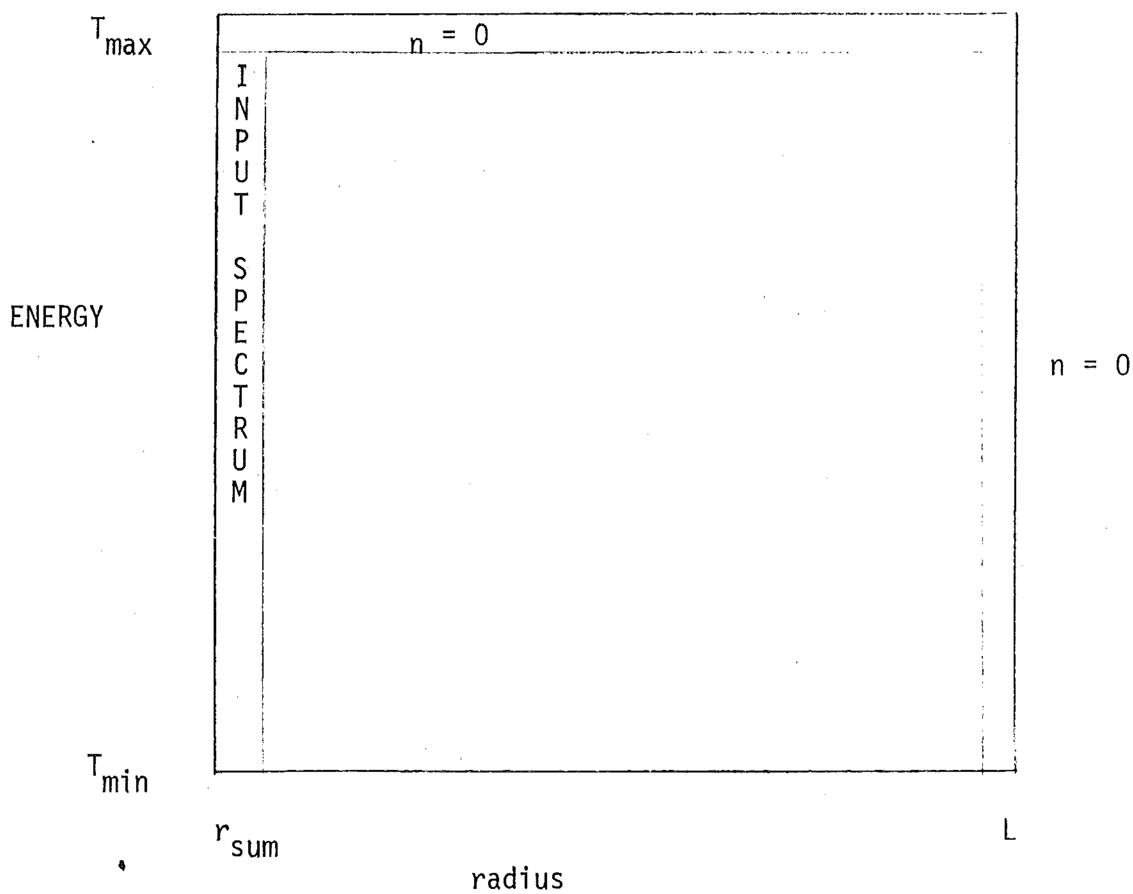
The remaining boundary condition is the energy spectrum at the sun. This is determined by choosing the energy spectrum of the input flux. Typical choices are shown in Cases I and II on page 3.

The boundary conditions for the differential equation are diagramed on the following page.

A change of variables, $u = r^{1/2}n$, is made, and the differential equation is solved using the Crank-Nicholson technique described by L. A. Fisk (JGR, 76, 221 (1971)). Crank-Nicholson is a numerical technique in which a grid of points is set up in radius-by-energy space and derivatives are approximated by finite differences. In the present project the grid is established using logarithmic steps in both energy and radius.

The injection rote at the sun was chosen to have an energy dependence of 1/T (where T is the kinetic energy) for energies between 1.0 MeV and

BOUNDARY CONDITIONS



T_c, and zero at other energies. T_c is then varied in order to investigate what energies particles have at earth when emitted with given energy at the sun. An example of this method has already been given on page 3.

Since the propagation equation is linear, a Green's function for the problem can be defined:

$$n(1AU,T) = \int G(T,T')F(T')\, dT'$$

where G is the Green's function, F is the spectrum of the particles injected at the sun, T refers to energy at earth, and T' refers to energy at the sun. In order to determine the Green's function, an approximate delta function is injected by cutting off the input spectrum at two different energies and determining the difference in the calculated steady-state desities at 1 AU as described in the example on page 2. Let T_1 be the cut-off energy for the first input spectrum and T_2 be the cut-off energy for the second. Let T_{AV} be the average of T_1 & T_2. Let n_1 and n_2 be the corresponding resulting densities at 1 AU. Then an approximation to the Green's function is given by:

$$G(T,T_{AV}) \simeq (n_2(T) - n_1(T))/\int_{T_1}^{T_2} F(T)\, dT$$

Then $G(T,T_{AV})$ is the contribution to the observed particle density at energy T at earth per particle input rate at T_{AV} at the sun. This function gives a clear indication of the amount of energy change taking place between the earth and the sun. An example of such a Green's function for particular values of the diffusion coefficient, solar wind velocity, and boundary distance is shown on page 12.

Instead of asking what energy particles have at earth if injected with a given energy, one may wish to know what is the distribution of energies that particles observed at a given energy at earth had at the sun. This is equivalent to viewing $G(T,T')$ as a function of T with T' fixed. Because the diffusion coefficient is assumed to be independent of energy, alpha is nearly independent of energy, and dT/dt is proportional to T for adiabatic energy change, there is a simple scaling relationship for the Green's function:

$$aG(aT,aT') = G(T,T')$$

This is because the shape of the distribution of energies seen at earth does not change when T' changes and each particle injected must go to some energy so that $G(T,T')\,dT$ is independent of T'. Thus if we know $G(1,T)$ (that is, the contribution to 1 MeV particles from energy T at the sun), we also know $G(1/T,1)$ (that is, what the energy distribution is at earth for particles emitted at 1 MeV at the sun).

The information about continuous emission obtained using the above techniques can be related to impulsive solar flares. The steady-state solution is effectively an integral of the contribution of particles injected in the past. More precisely, continuous emission can be devined as the sum of an infinite number of discrete impulsive flares (as in a Riemann Integral). Let $R(r,t,T)$ be the solution to the time-dependent Fokker-Planck propagation equation for the impulsive emission of a single particle at the sun. Since the problem is linear, $N \cdot R(r,t,T)$ is the solution for the injection of N particles at the sun. Then the solution for continuous injection is given by:

$$n(r,T) = N \sum_{j=1}^{\infty} R(r, j\Delta t,T);$$ where N is the number of particles injected for each flare (N is constant), and Δt is the time between flares (also a constant).

Let $y = N/\Delta t$ be the constant injection rate. Then

$$n(r,T) = y \int_0^{\infty} R(r,t',T)dt'$$

But this is just the time integral of the solution to one impulsive solar flare. Thus we have

$$[\text{density/input rate}]_{\text{cont.inj.}} = [\int_0^{\infty} \text{ndt/number injected}]_{\text{flare}}$$

Using this equation, the energy change for continuous emission can be related to the energy change for the time-integral of densities of flares.

RESULTS:

The above procedures have been applied to the sets of parameters listed below. The resulting Green's functions are shown on the pages listed. $\overline{T}$ is defined as follows:

$$\overline{T} \equiv \int_1^\infty T'G(1,T')dT' / \int_1^\infty G(1,T')dT'$$

Thus $\overline{T}$ is the average energy 1 MeV particles had at the sun assuming the same number of particles were injected at every energy.

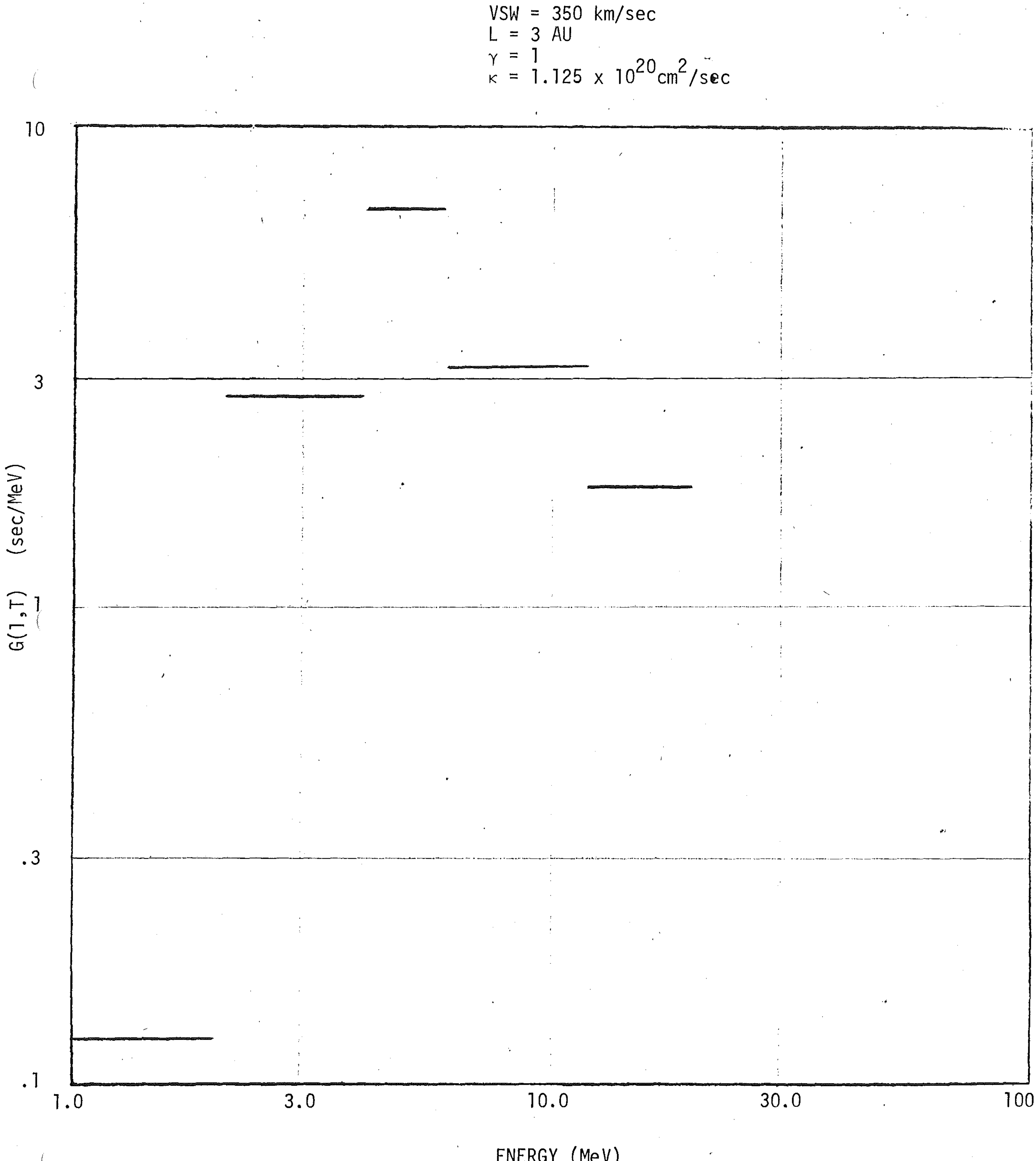
Case	Page	VSW km/sec	L AU	Kappa $10^{20}cm^2/sec$	$\overline{T}$
3	12	350	3	1.125	5
4	13	350	3	5	2
5	14	350	3	5	(this is the inversion of Case 4)
6	15	350	5	5	2
7	16	350	3	15	1.2
8	17	350	3	5r/AU	10?

The chart shows that typical values of the average energy change is about a factor of 2. For large values of the diffusion coefficient the particles quickly escape the sun, where the rate of energy change is high, and thus lose little energy as shown in Case 7. The converse is true for low values of the diffusion coefficient as shown in Case 3. Changing the value of the boundary distance has little effect since most of the energy is lost before the particles feel the effect of the boundary as shown by comparing Case 6 and Case 4. Changing the radial dependence of the diffusion coefficient can have a significant effect on the amount of energy change as shown in Case 8. However, it is difficult to determine what reasonable values for kappa are for arbitrary dependence on radius. The obvious value to keep constant is the time-to-maximum (see Palmer) but it is not known how to compute time-to maximum including energy change. Energy change can be too large a contribution to be ignored as Palmer did. The amount of energy lost should also be proportional to the solar wind velocity since dT/dt is proportional to VSW and the time to escape the sun depends mostly on diffusion.

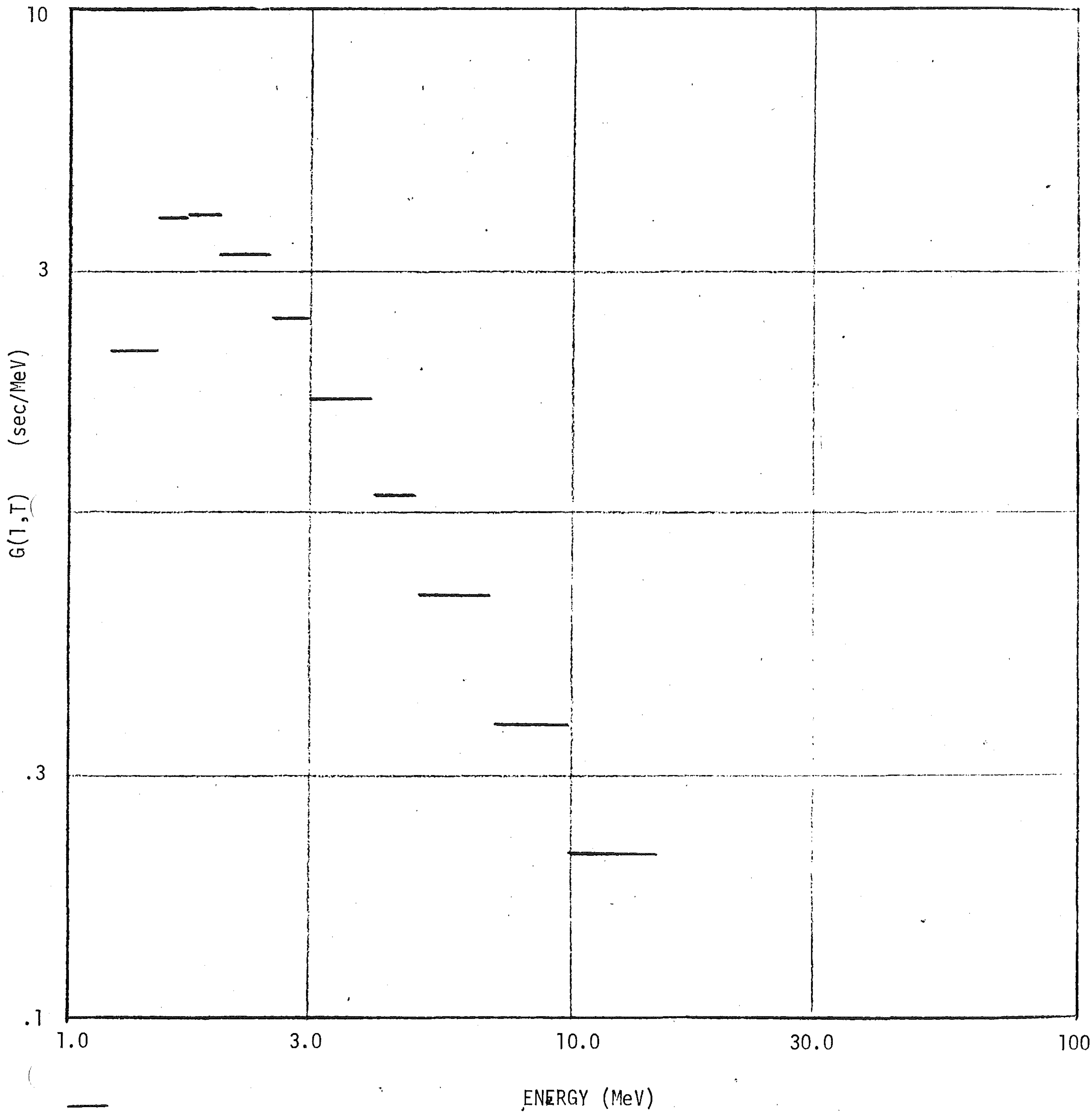
CHECKS:

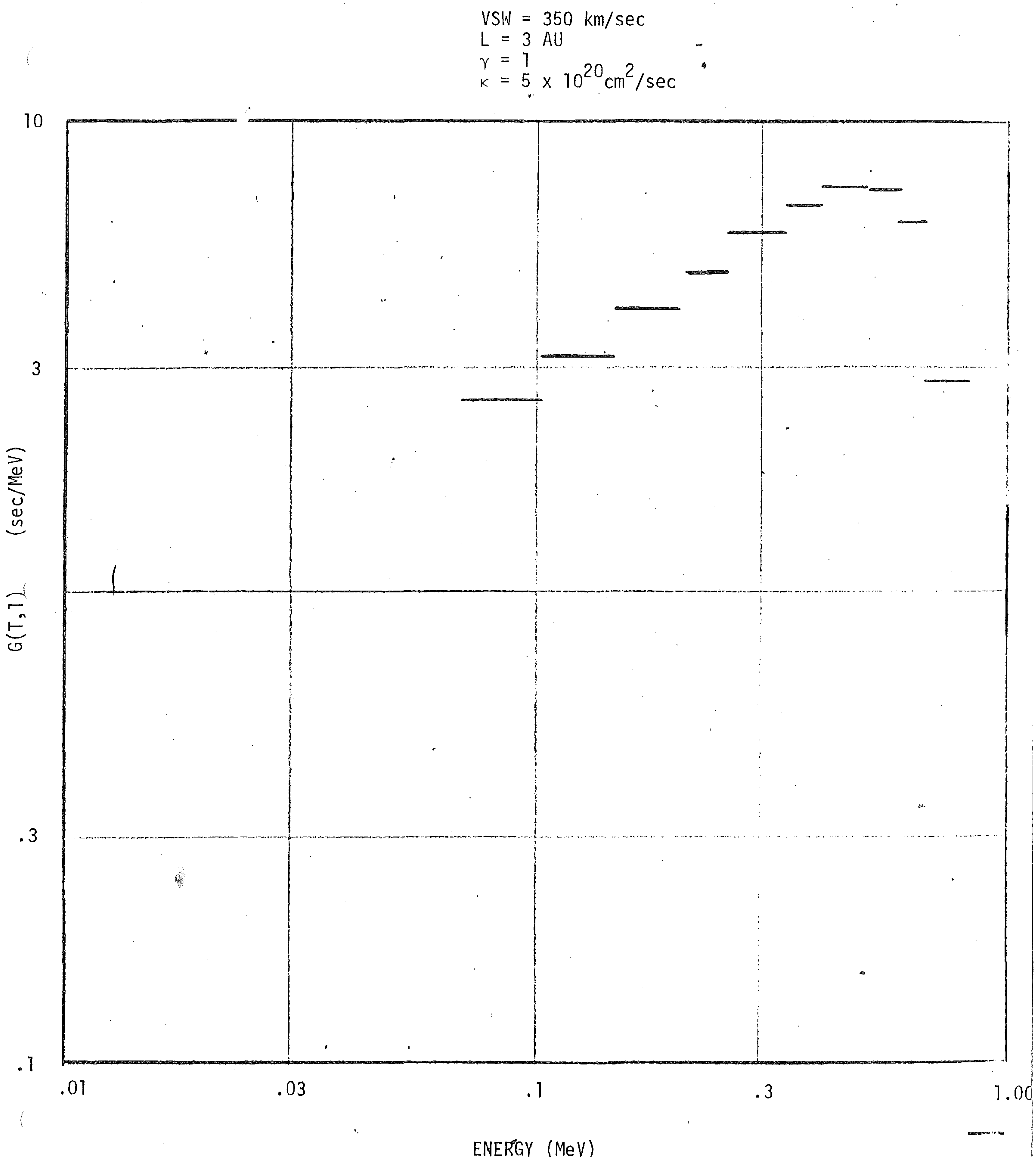
Because the solutions are numerical, checks have been made to see if they are reasonable. This was done for the Case 3. This sample was compared to the time-integral of the density computed by the impulsive solution developed by Lupton and Stone; the answers agreed to within 2%. The grid size was found to have an effect on the solution. The number of grid points was increased until increasing the number did not effect the solution.

Finally the results are comparable to those obtained by A. J. Owens, Urch and Gleeson, and in general terms Palmer.

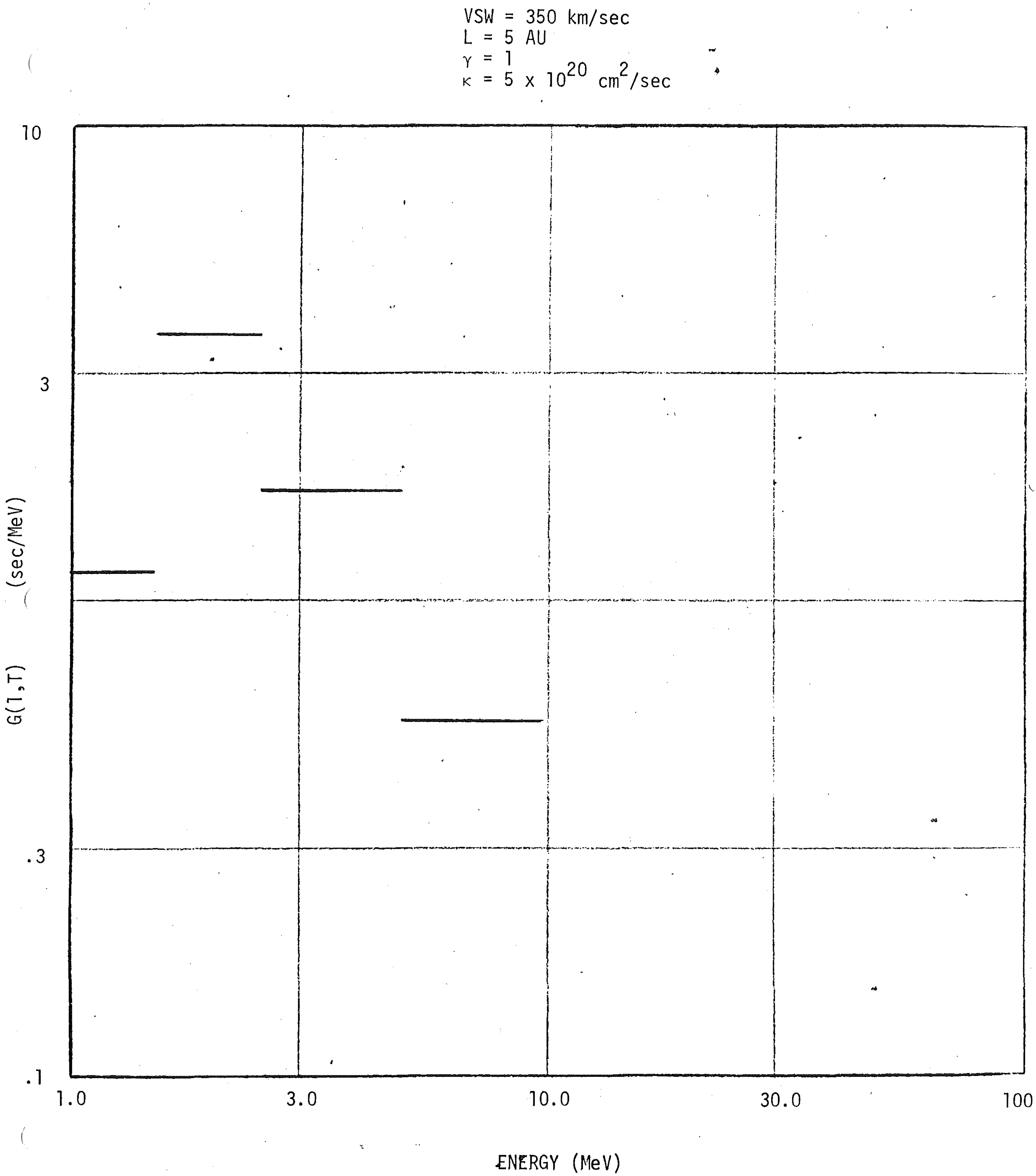

VSW = 350 km/sec
L = 3 AU
γ = 1
κ = 1.125 x 10^20 cm^2/sec
10
3
1
.3
.1
G(1,T) (sec/MeV)
1.0
3.0
10.0
30.0
100
ENERGY (MeV)

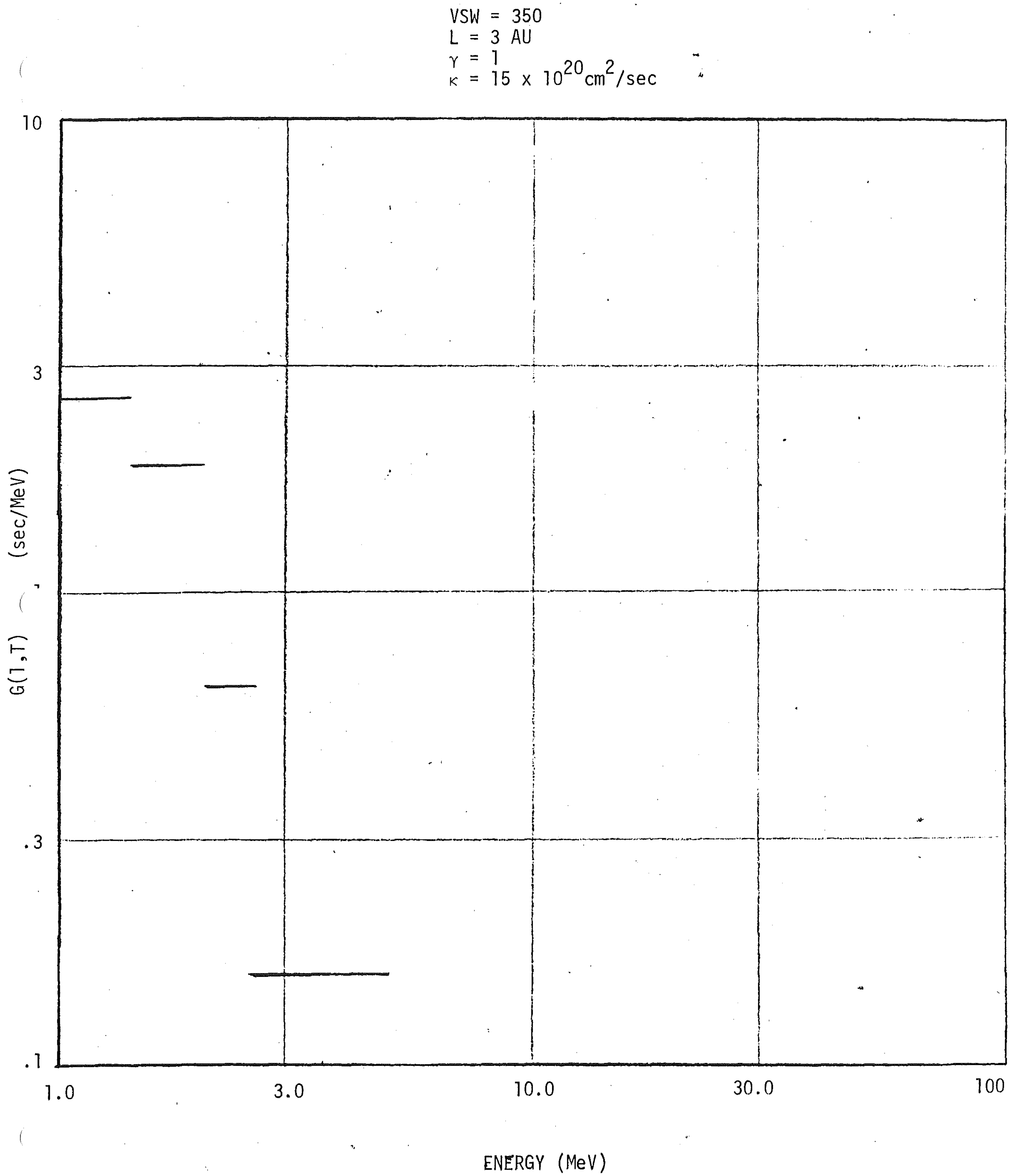
VSW = 350 km/sec
L = 3 AU
$\gamma = 1$
$\kappa = 5 \times 10^{20} cm^2/sec$





VSW = 350 km/sec
L = 3 AU
γ = 1
κ = 5 x 10^20 cm^2/sec
10
3
.3
.1
G(T,1) (sec/MeV)
.01
.03
.1
.3
1.00
ENERGY (MeV)


VSW = 350 km/sec
L = 5 AU
γ = 1
κ = 5 x 10^20 cm^2/sec
10
3
1
.3
.1
G(1,T) (sec/MeV)
1.0
3.0
10.0
30.0
100
ENERGY (MeV)


VSW = 350
L = 3 AU
γ = 1
κ = 15 x 10^20 cm^2/sec
10
3
1
.3
.1
G(1,T) (sec/MeV)
1.0
3.0
10.0
30.0
100
ENERGY (MeV)

VSW = 350 km/sec
L = 3 AU
γ = 1
$\kappa = 5 \times 10^{20} cm^2/sec \cdot R/AU$