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INTERNAL REPORT #81

ENERGY LOSS FLUCTUATIONS IN THICK ABSORBERS

by

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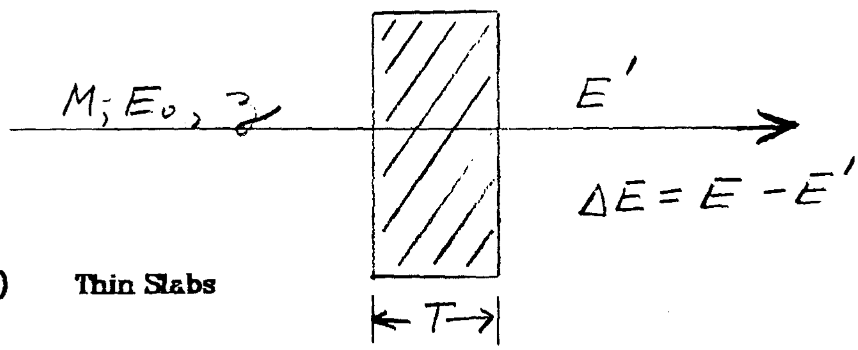
I) Basic Ideas- see Rossi (1952)



a) Thin Slabs

When traversing a slab of material, ions mainly lose energy through Coulomb collisions with the electrons in the slab. To calculate the r.m.s. deviation in the energy loss, $\sigma_{\Delta E}$, we start with:

$$\sigma_{\Delta E}^2 = <(\Delta E)^2> - <\Delta E>^2.$$

For a thin slab of thickness ΔT:

$$<\Delta E> \approx \Delta T \frac{dE}{dX}$$

$$<(\Delta E)^2> \approx \Delta T \frac{d(E^2)}{dX}$$

$$\sigma_{\Delta E}^2 = \Delta T \frac{d(E^2)}{dX} - (\Delta T)^2 \left(\frac{dE}{dX}\right)^2$$

$$= \Delta T \frac{d(E^2)}{dX} \textit{ first order.}$$

For the scattering of spin 1/2 particles by heavy spin 0 ions of charge z,

Φ = *probability of an energy loss* ΔE *in* $d(\Delta E)dX$.

$$\Phi(E,\Delta E) = \frac{2\pi NZ'z^2e^4}{mc^2\beta^2}\frac{d(\Delta E)}{(\Delta E)^2}\left[1 - \beta^2\frac{\Delta E}{\Delta E_{\max}}\right]dX$$

Here: N = number density of slab material

Z' = charge of slab nuclei

z = ion charge

m = electron mass

$\Delta E_{\max} = 2mc^2\,\beta^2\,\gamma^2$

We can now calculate $\frac{dE^2}{dX}$:

$$\frac{d(E^2)}{dX} = \int_0^{\Delta E_{\max}} (\Delta E)^2\,\Phi(E,\Delta E)\,d(\Delta E)$$

$$\frac{d(E^2)}{dX} = 4\pi NZ'z^2e^4\gamma^2\left(1 - \frac{\beta^2}{2}\right)$$

$$= \sigma_0^2\gamma^2\left(1 - \frac{\beta^2}{2}\right) = \sigma_0^2\,h(\beta) = \sigma_\beta^2$$

$$\sigma_0^2 = z^2\,\frac{2Z'}{A}\,(\,.280\ Mev\,)^2/\ (g/cm^2)$$

$$= z^2(\,.135\ MeV\,)^2/\ mm \qquad (Silicon)$$

$$= z^2(\,.427\ MeV\,)^2/\ cm \qquad (Silicon)$$

For a thin slab we get the result:

$$\sigma_{\Delta E}^2 = \sigma_\beta^2\,\Delta T$$

Then for a finite thickness slab, we just add $\sigma_{\Delta E}^2$ from each ΔT in quadrature and get the result derived by Bohr in 1915,

$$\sigma^2_{\Delta E} = \sigma^2_0 \; T.$$

For a thin slab, fluctuations in energy loss for the ion arise essentially from fluctuations in the number of electrons that hit the ion. If each collision caused the ion to lose the same amount of energy, then $\sigma^2_{\Delta E}$ would be proportional to N, where N is the number of electron collisions. Thus to first order, $\sigma^2_{\Delta E}$ should be proportional to T, as the mathematics shows. This is the "stochastic" contribution to the energy loss fluctuations.

b) Thick slabs

For thick slabs, two effects make it necessary to improve the thin slab result. First, $\Phi(E_0, \Delta E)$ depends on E and must be suitably averaged over the particle energy loss history in the slab.

$$\sigma^2_{\Delta E} = \int_0^T \sigma^2_0 \,(E(T))\, dT$$

The second effect is important where the slope of the dE/dX function is large and negative, and causes a systematic spread in the energy loss distribution. If we focus on a case where a distribution of finite width enters a slab, the particles at higher energy lose less energy than those at lower energy, causing a systematic increase in the width of the energy loss distribution. The region where the energy loss distribution width is dominated by this effect is called the "bulk" region.

As will be seen later, the energy distribution at a pathlength S from a beam at fixed energy E_0, F(E,S), is approximately Gaussian. When this is true, a formula first derived by Symon (concisely sumarized by Payne (1969)) can be used to calculate $\sigma^2_{\Delta E}$

in terms of the first two moments of the energy loss probability function Φ.

$$\sigma^2_{\Delta E}(T) = 2\int_{E'}^{E_0} \frac{M_2(x)}{[\,M_1(x)]^3}[M_1(E')]^2\,dx$$

$$+ \left[\frac{M_1(E')}{M_1(E_0)}\right] \sigma^2_{\Delta E}(0)$$

Where: $$M_j(E) = \frac{1}{(j)!} \int_0^{\Delta E_{max}} (\Delta E)^j\ \Phi(E,\Delta E)\ d(\Delta E)$$

and $\sigma^2_{\Delta E}(0)$ = the second moment of the energy distribution before entering the slab.

I will derive two approximations to this result and evaluate it numerically, always for the case where $\sigma^2_{\Delta E}(0) = 0$.

II) Quantitative Theory

a) Bohr approximation - ignores the slowing down of the ion

Here, $\dfrac{M_1(E')}{M_1(x)} \approx 1$ and M_2 is taken to be a constant.

$$\sigma^2_{\Delta E} = 2M_2 \int \frac{1}{M_1(x)}\,dx \;=\; 2M_2 \int \frac{dE}{(dE/dX)}$$

$$\sigma^2_{\Delta E} = 2M_2 \int_{R'}^{R_0} dR \;=\; 2M_2\,(R_0 - R') \;=\; 2M_2\,T$$

$$\sigma^2_{\Delta E} = \sigma_0^2\,h(\beta)\,T = \sigma_B^2\,T$$

b) Power law approximation to dE/dX.- G. Hurford, M. Wiedenbeck

Approximate dE/dX by a power law = $CE^{-\lambda+1}$ and let $M_2 \approx$ constant.

Let R = range of a particle of energy E_o.

$$\sigma^2_{\Delta E} = 2M_2\,(E')^{2-2\lambda} \int_{E'}^{E_o} \frac{1}{C}\, x^{3\lambda-3}\, dX$$

$$\sigma^2_{\Delta E} = \sigma^2_B\; T\, D^2$$

$$D^2 = \frac{\lambda}{3\lambda-2}\,\frac{R}{T}\left[(1-\frac{T}{R})^{\frac{2}{\lambda}-2} - (1-\frac{T}{R})\right]$$

For $T = \varepsilon R,\ \frac{R}{T} = \frac{1}{\varepsilon}$

$\lim\limits_{\varepsilon\to 0} D^2(\varepsilon) = 1$ and we get the thin slab result.

c) Numerical Evaluation in dimensionless variables

$$M_1 = \frac{dE}{dX} = z^2\, g(\beta)\left[\frac{\sigma_0^2}{mc^2}\right]$$

$$\sigma^2_{\Delta E} = (\,mc^2)^2\, g^2\,(\beta') \int_{E'}^{E_o} \frac{h(\beta)}{g^3(\beta)}\,(\,dE/\,mc^2\,)$$

Define a range-like function:

$$B(E_0) = \frac{mc^2}{\sigma_0^2} \int_0^{E_0} \frac{h(\beta)}{g^3(\beta)}\, dE$$

Then:

$$\sigma_{\Delta E}^2 = D^2\, \sigma_B^2\, T,$$

with:

$$D^2 = \frac{B(E_0) - B(E')}{R(E_0) - R(E')}\, \frac{g^2(\beta')}{h(\beta)}.$$

III) Numerical examples

For the numerical work in sec IV), two forms of dE/dX were chosen, as explained below.

a) Analytical dE/dX with z^2 scaling

$$\frac{dE}{dX} = z^2\, \frac{\sigma_0^2}{mc^2}\, \frac{1}{\beta^2} \Big\{ \ln\Big(\frac{2mc^2}{I}\Big)$$

$$- \ln(\beta^2) - \ln(1 - \beta^2) - \beta^2 \Big\}$$

Using I = 170 eV, this reproduces the Janni dE/dX for Silicon with < 1.5% error for 1000 MeV/n > E > 10 MeV and < 4% error for E > 1 MeV/n.

b) Barkas and Berger effective charge

Here we assume that dE/dX is as given by sec IIIa) but that the charge is the velocity dependent charge as given by Barkas and Berger (1963).

Z = nuclear charge of the ion

Z^{*} = ionic charge

$$\left(\frac{Z}{Z^{*}}\right)^{2} - 1 = q(\beta, Z) = 1.3E{-5}\,\frac{Z^{\frac{5}{3}}}{\beta^{\frac{7}{3}}}$$

$$q(\beta, Z) = \frac{1.258}{Z}\left(\frac{Z}{137\beta}\right)^{\frac{7}{3}}$$

$$\left.\frac{dE}{dX}\right|_{Barkas} = \frac{1}{1+q}\left.\frac{dE}{dX}\right|_{stripped}$$

IV) Comparison of Results

Figure 1 shows the D factor plotted vs. (R/T - 1) for several methods of calculation. The D factors were calculated in the power law approximation, with λ = 1.77, and by numerical calculation with the section IIIa) and IIIb) forms for dE/dX.

The first thing to notice is that the power law approximation to the D factor is in excellent agreement with the stripped (IIIa) numerical D factor. This occurs because most of the contribution to the D factor integral occurs just as the particle enters the slab. If a power law approximation to dE/dX works well there, then the power law D factor will be useful and accurate. Even the numerical D factor with the Barkas and Berger stripping correction hardly differs from the numerical stripped and power law forms, except for high charges and fairly thin slabs. Only for T = 100μ and T = 250μ with iron ions would the new calculation of the D factor have any significant bearing on the expected $\sigma_{\Delta E}^{2}$ for R/T significantly greater than 1.

V) Limits for the Validity of the Calculation

The approximation used in the above calculations assumed that the distribution function in energy for initially mono-energetic ions would be essentially Gaussian after traversing a slab of thickness T. The range of energies over which this assumption holds can be checked with numerical calculations in two papers by Tschlar (1968a,b).

The 1968b paper treats ions correctly for high ΔE. The deviation of the energy distribution from a Gaussian form is parameterized by the skewness, sk, given by $\frac{T_m - T_{-\frac{1}{2}}}{T_{+\frac{1}{2}} - T_m}$ with T_m equal to the energy of the maximum of the energy loss distribution and $T_{\pm\frac{1}{2}}$ equal to the energies of the upper and lower half height points of the energy loss distribution.

The 1968a paper treats ions with low ΔE. Here the deviations from Gaussian behavior are measured by the dimensionless third moment of the energy loss distribution, $\gamma_3{}^2$.

$$\gamma_3 = \frac{A_3}{\sigma_{\Delta E}^3}$$

$$A_3 = \int (E - \langle E \rangle)^3 \, F(E,S) \, dE$$

For comparison, a distribution with $\gamma_3^2 = 0.1$ has sk ~ 1.08.

The qualitative results show the energy loss distribution as Gaussian except for very large and very small ΔE. For a few special cases I list, in Table 1, some energy limits obtained at sk = 1.15 at high ΔE and $\gamma_3^2 = 0.1$ at low ΔE, between which the energy loss distribution is, for all practical purposes, Gaussian.

TABLE 1

Particle	**Energy**	**max**(E'/E_0)	**min**(E'/E_0)
proton	78 Mev	92.5%	22%
proton	7.8 Mev	92.5%	18%
alpha	78 Mev/n	98.2%	13.5%
alpha	7.8 Mev/n	98.2%	10.0%
^{20}Ne	78 Mev/n	99.6%	8.0%
^{20}Ne	7.8 Mev/n	99.75%	4.6%
^{100}Ru	78 mev/n	< 99.8%	4.8%
^{100}Ru	7.8 Mev/n	< 99.8%	never exceeds sk=1.15

VI) References

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FIGURE 1