A Technique for Determining Particle Anisotropies using the Voyager Low Energy Telescopes

by

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Abstract

In regions of space where there is no apriori information about the orientation of possible particle anisotropies, the four Voyager Low Energy Telescopes (LETs) can be used to specify the direction and magnitude of any first-order anisotropy. If directions in space can be identified that represent symmetry axes for particle motion (e.g. magnetic field lines or an observer's direction of motion with respect to a particle population), the LETs can be used to determine the magnitudes of higher-order, as well as first-order, anisotropies about these directions. The coefficients of the linear equations that relate the anisotropy magnitudes to the telescope counting rates are calculated in this report.
The counting rate of a particle telescope in a flux \( F(\theta, \varphi, E) \) is

\[
C = \int_{E_1}^{E_2} dE \int d\Omega \, F(\theta, \varphi, E) \, A(\theta, \varphi)
\]

where \( E_1 - E_2 \) is the energy range of particles whose energy loss in the detectors exceeds the counting thresholds and \( A(\theta, \varphi) \) is the effective area of the telescope (e.g. the overlap area of two detectors). For an isotropic flux, \( j(E) \),

\[
C = \int_{E_1}^{E_2} dE \int d\Omega \, A(\theta, \varphi) = A\Omega \int_{E_1}^{E_2} dE \, j(E)
\]

where \( A\Omega = \int d\Omega A(\theta, \varphi) \) is the telescope geometry factor. For an anisotropic flux, one would like to use the counting rates in two or more independent telescopes to obtain information about the angular dependence of \( F(\theta, \varphi, E) \) or, assuming \( F \) is separable as \( F(\theta, \varphi, E) = I(\theta, \varphi) j(E) \), about \( I(\theta, \varphi) \) The specific application considered in this report is the four LETs that are part of the Cosmic Ray Subsystem on the Voyager spacecraft. The approach here will be to expand \( I(\theta, \varphi) \) in a series of spherical harmonics and do the angular integration in equation 1 numerically for each term of the series using the appropriate formula for \( A(\theta, \varphi) \). The resulting linear equations can then be solved to give the coefficients in the spherical harmonic expansion in terms of the four counting rates.

Since the spherical harmonics \( Y_{lm}(\theta, \varphi) \) are a complete set of functions, any function \( I(\theta, \varphi) \) (with sufficient continuity properties) can be expressed as a series in terms of them,

\[
I(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} \, Y_{lm}(\theta, \varphi)
\]

where the \( a_{lm} \) are constants. The LETs give four independent equations
that can be used to determine four of the constants. Given no additional information, we chose to determine the first four \( a_{lm} \). The expansion is most convenient in terms of the spherical harmonics defined by Morse and Feshbach (1953, p. 1271):

\[
I(\psi, \varphi) = a_{00} + a_{10} \cos \psi + a_{11} \sin \psi \cos \varphi + a_{1-1} \sin \psi \sin \varphi .
\]

This equation can be put into a simpler form by rotating the coordinate system by the Euler angles (Goldstein, 1950, p. 107) \( \psi_{i1} \), \( \varphi_{i1} \), and \( \psi_{i1} \) such that

\[
\begin{align*}
\cos \psi_{i1} & = \frac{a_{10}}{\sqrt{a_{10}^2 + a_{11}^2 + a_{1-1}^2}} \\
\sin \psi_{i1} & = \frac{\sqrt{a_{11}^2 + a_{1-1}^2}}{\sqrt{a_{10}^2 + a_{11}^2 + a_{1-1}^2}} \\
\cos \varphi_{i1} & = -\frac{a_{1-1}}{\sqrt{a_{11}^2 + a_{1-1}^2}} \\
\sin \varphi_{i1} & = \frac{a_{11}}{\sqrt{a_{11}^2 + a_{1-1}^2}} \\
\psi_{i1} & = \text{arbitrary (choose } 0) .
\end{align*}
\]

The result is

\[
I(\psi, \varphi) = a_0 + a_1 \cos \psi_1
\]

where \( a_0 = a_{00} \), \( a_1 = \sqrt{a_{10}^2 + a_{11}^2 + a_{1-1}^2} \) and \( \psi_1 \) is the angle relative to the new \( z \)-axis. This form shows explicitly that the LETs can measure the magnitude and direction of a Compton-Getting-type anisotropy. The four parameters determined in this case are \( a_0, a_1, \psi_{i1}, \) and \( \varphi_{i1} \).

In some environments, reasonable guesses can be made for the values of one or more of these constants and the LETs can then be used to specify the parameters of higher order spherical harmonics. For instance, in a magnetosphere one might assume that the only first-order anisotropy is due to the corotation of the particle distribution past the observer. In this case, the direction of the axis of the anisotropy (\( \psi_{i1} \) and \( \varphi_{i1} \)) is known, and the extra two degrees of freedom can be used to determine \( \cos^2 \psi \) terms that pertain to pitch-angle distributions about the field lines.

If it is assumed that each higher order spherical harmonic term is axially symmetric about some axis, \( \hat{e}_i \), it can be shown (Sanderson and
Hynds, 1977, and references therein) that \( I(\psi, \varphi) \) can be represented as:

\[
I(\psi, \varphi) = a_0 + a_1 \cos \psi_1 + a_2 \frac{3}{2} (\cos^2 \psi_2 - \frac{1}{3}) + a_3 \frac{5}{2} (\cos^3 \psi_3 - \frac{3}{5} \cos \psi_3) + \cdots \tag{3}
\]

where \( \psi_1, \psi_2 \) and \( \psi_3 \) are measured with respect to \( \hat{e}_1, \hat{e}_2, \) and \( \hat{e}_3 \). There are clearly fewer parameters in the \( \cos^2 \psi \) and \( \cos^3 \psi \) terms in this equation than in the same order terms in the spherical harmonic equation. For the \( \cos^2 \psi \) term, there are five spherical harmonic coefficients \( a_{20}, a_{21}, a_{2-1}, a_{22}, \) and \( a_{2-2} \), but in equation 3 only three parameters appear; namely, \( a_2 \) and the Euler angles \( \psi_{e2} \) and \( \varphi_{e2} \) of the \( \hat{e}_2 \) axis. Similarly, for the \( a_3 \) term, the seven spherical harmonic coefficients correspond to the three parameters \( a_3, \psi_{e3}, \) and \( \varphi_{e3} \) above. The reason is that, to be written in the form of equation 3, the \( a_{2m} \) and \( a_{3m} \) terms in the spherical harmonic expansions were assumed to be axially symmetric about some axes \( \hat{e}_2 \) and \( \hat{e}_3 \). The Maxwell theory of poles of the spherical harmonics (see e.g. Sanderson and Hynds, 1977), from which equation 3 was derived, requires, in general, two axes \( \hat{e}_{2a} \) and \( \hat{e}_{2b} \) to represent the \( a_{2m} \) spherical harmonic terms. There are therefore five parameters, \( a_{2}, \psi_{e2a}, \varphi_{e2a}, \psi_{e2b} \) and \( \varphi_{e2b} \) to match the five \( a_{2m} \). For the \( a_{3m} \) terms, three axes (six Euler angles) are required. The axial symmetry assumption means that two \( a_2 \) axes line up and that the three \( a_3 \) axes line up. The expansion for \( I(\psi, \varphi) \) given in equation 3 should therefore only be used if there is some physical basis for assuming axially symmetric \( \cos^2 \psi \) or \( \cos^3 \psi \) distributions. In the previous magnetosphere example, the field lines impose a natural symmetry condition on the pitch angle distributions.

The next step is to perform the angular integration in equation 1. For \( F(E, \psi, \varphi) = j(E) I(\psi, \varphi) \),

\[
C = \int_{E_1}^{E_2} dE j(E) \int_0^{2\pi} d\varphi \int_0^\pi d\psi A(\psi, \varphi) [a_0 + a_1 \cos \psi_1 \nonumber + a_2 \frac{3}{2} (\cos^2 \psi_2 - \frac{1}{3}) + a_3 \frac{5}{2} (\cos^3 \psi_3 - \frac{3}{5} \cos \psi_3)] \tag{4}
\]

In order to actually do the integrals \( \psi_i \) must be transformed into \( \psi \) and \( \varphi \) using

\[
\psi_i = a_i (\psi - \psi_{e2}),
\psi = \frac{3}{2} a_1 \psi_1, \psi_1 = \psi_2, \psi_2 = \psi_3, \psi_3 = \psi.
\]
\[
\cos \theta_i = \cos \theta_i \cos \alpha_i + \sin \theta_i \sin \alpha_i \cos (\varphi - \beta_i) \quad i = 1, 2, 3, \ldots \tag{5}
\]

where \(\alpha_i\) and \(\beta_i\) are the angles of the \(i^{th}\) symmetry axis \((\hat{e}_i)\) in the telescope coordinate frame \(\text{(z axis = telescope symmetry axis)}\).

It is shown in the Appendix that, as one would expect from the cylindrical symmetry, the integral does not depend on \(\beta_i\). We also need an explicit expression for \(A(\psi, \varphi)\). For the LET case the area-overlap formula for two separated circular disks can be used:

\[
A(\psi, \varphi) = \pi r^2 \cos \psi
\]

\[
= \cos \psi \left[ \frac{r_1^2}{2} \left[ 2\phi_1 - \sin 2\phi_1 \right] + \frac{r_2^2}{2} \left[ 2\phi_2 - \sin 2\phi_2 \right] \right]
\]

\[
= 0 \quad 0 \leq \psi < \psi_c
\]

\[
= 0 \quad \psi_c \leq \psi < \psi_m
\]

\[
= 0 \quad \psi_m \leq \psi
\]

(Sullivan, 1971) where:
\[
\begin{align*}
\mathbf{r}_1 &= \text{radius of detector 1} \\
\mathbf{r}_2 &= \text{radius of detector 2} \\
x_s &= \text{smaller of } r_1 \text{ and } r_2 \\
\psi_c &= \tan^{-1} \left( \frac{|r_1 - r_2|}{d} \right) \\
\psi_m &= \tan^{-1} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{d} \right) \\
\Phi_1 &= \cos^{-1} \left( \frac{\mathbf{r}_1^2 + d^2 \tan^2 \vartheta - r_2^2}{2d\mathbf{r}_1 \tan \vartheta} \right) \\
\Phi_2 &= \cos^{-1} \left( \frac{\mathbf{r}_2^2 + d^2 \tan^2 \vartheta - r_1^2}{2d\mathbf{r}_2 \tan \vartheta} \right) \\
d &= \text{detector separation}
\end{align*}
\]

Note that due to the assumed cylindrical symmetry, \( A(\psi, \varphi) \) does not depend on \( \varphi \).

The integrals can now be done. A convenient way to express the answer is to define

\[
K_0 = 2\pi \int_0^\pi d\psi A(\psi) \sin \psi = A\Omega
\]

\[
K_1(\alpha_1) = \int_0^{2\pi} d\varphi \int_0^\pi d\psi A(\psi) \cos \psi_1 \sin \psi
\]

\[
K_2(\alpha_2) = \int_0^{2\pi} d\varphi \int_0^\pi d\psi A(\psi) \frac{3}{2} \left( \cos^2 \psi_2 - \frac{1}{3} \right) \sin \psi
\]

\[
K_3(\alpha_3) = \int_0^{2\pi} d\varphi \int_0^\pi d\psi A(\psi) \frac{5}{2} \left( \cos^3 \psi_3 - \frac{3}{5} \cos \psi_3 \right) \sin \psi
\]

Comparing equations 4 and 7, we see that the counting rate in a telescope can now be succinctly expressed as
In the case of the LETs there are four such equations so that in principle \( a_0, a_1, a_2, \) and \( a_3 \) can be determined. In the Appendix, the values of \( K_0, \) \( K_1, \) \( K_2, \) and \( K_3 \) are calculated for both a window-L1 and a L1-L2 configuration. The results are listed in the computer output at the end of this report. As is indicated in equation 8 and shown in the Appendix, the values of \( K_i \) depend on the angles \( \alpha_i \) between the telescope axis and the \( i^{th} \) symmetry axis. Fortunately, however, the \( \alpha_i \) dependence comes out of the integrals, so that the integrals only need be done once for a given detector configuration. For example, using the numbers listed in the computer output, the fully expanded expression for the counting rate for the nominal L1-L2 configuration \((r_1=r_2=0.95 \text{ cm}, d=4.08 \text{ cm})\) is

\[
C = \int_{E_1}^{E_2} dE \, j(E) \sum_{i=0}^{n} a_i K_i(\alpha_i).
\]

\[
C = \int_{E_1}^{E_2} dE \, j(E) \left[ a_0 \cdot 0.4367 + a_1 \cdot 0.4263 \cdot \cos \alpha_1 + \right.
\]
\[
\left. a_2 (0.6091 \cdot \cos^2 \alpha_2 - 0.2030) + a_3 (0.9432 \cdot \cos^3 \alpha_3 - 0.5659 \cdot \cos \alpha_3) \right].
\]
Appendix

In this section the equations for $K_i$ (equations 7) are put into forms that can be numerically integrated given the detector configuration. First, since $A(\psi)$ does not depend on $\varphi$, all integrals involving $\varphi$ can be done analytically. A useful equation is

$$\int_0^{2\pi} d\varphi \cos^n(\varphi - \beta_i) = \int_0^{2\pi} d\varphi \cos^n \varphi = \begin{cases} 2\pi & n = 0 \\ 0 & n \text{ odd} \\ \pi & n = 2, 4, 6, \ldots \end{cases}$$

As expected from the cylindrical symmetry, the $\beta_i$ dependence drops out so that $\beta_i$ can be set to zero in equation 5.

$K_0$:

$$K_0 = 2\pi \int_0^{\pi} d\psi A(\psi) \sin \psi$$

$K_1(\alpha_1)$:

$$\cos \psi_1 = \cos \psi \cos \alpha_1 + \cos \varphi \text{ terms}$$

$$K_1(\alpha_1) = \cos \alpha_1 2\pi \int_0^{\pi} d\psi A(\psi) \cos \psi \sin \psi$$

$K_2(\alpha_2)$:

$$\cos^2 \psi_2 = \cos^2 \psi \cos^2 \alpha_2 + \sin^2 \psi \sin^2 \alpha_2 \cos 2\varphi + \cos \varphi \text{ terms}$$

$$= \cos^2 \alpha_2 \left[ \cos^2 \psi - \sin^2 \psi \cos 2\varphi \right] + \sin^2 \psi \cos 2\varphi + \cos \varphi \text{ terms}$$

$$K_2(\alpha_2) = \frac{3}{2} \cos^2 \alpha_2 \left[ 2\pi \int_0^{\pi} d\psi A(\psi) \cos^2 \psi \sin \psi - \int_0^{2\pi} d\varphi \cos^2 \varphi \int_0^{2\pi} d\psi A(\psi) \sin^2 \psi \sin \psi \right]$$

$$+ \frac{3}{2} \int_0^{2\pi} d\varphi \cos^2 \varphi \int_0^{2\pi} d\psi A(\psi) \sin^2 \psi \sin \psi - \frac{1}{2} 2\pi \int_0^{2\pi} d\psi A(\psi) \sin \psi$$
\[
\begin{align*}
&= \frac{3\pi}{2}\cos^2 \alpha_2 \left[ 2\int_0^\pi d\vartheta A(\vartheta)\cos^2 \vartheta \sin \vartheta - \int_0^\pi d\vartheta A(\vartheta)\sin^3 \vartheta \right] \\
&\quad + \pi \left[ \frac{3}{2}\int_0^\pi d\vartheta A(\vartheta)\sin^3 \vartheta - \int_0^\pi d\vartheta A(\vartheta)\sin \vartheta \right] \\
K_3(\alpha_3): \\
\cos^3 \vartheta_3 &= \cos^3 \vartheta \cos^3 \alpha_3 + 3\cos \vartheta \cos \alpha_3 \sin^2 \vartheta \sin^2 \alpha_3 \cos^2 \varphi + \text{odd powers of } \cos \varphi \\
&= \cos^3 \alpha_3 (\cos^3 \vartheta - 3\cos \vartheta \sin^2 \vartheta \cos^2 \varphi) + \\
&\quad \cos \alpha_3 \cos \vartheta \sin^2 \vartheta \cos^2 \varphi + \text{odd powers of } \cos \varphi
\end{align*}
\]

\[K_3(\alpha_3) = \frac{5}{2}\cos^3 \alpha_3 \left[ 2\pi \int_0^\pi d\vartheta A(\vartheta)\cos^3 \vartheta \sin \vartheta - 3\int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \cos^2 \varphi \int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \sin^3 \vartheta \right] \\
+ \cos \alpha_3 \left[ \frac{3}{2} \frac{2\pi}{2} \int_0^\pi d\vartheta \cos^2 \varphi \int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \sin^3 \vartheta - 2\pi \frac{3}{2} \frac{\pi}{2} \int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \sin \vartheta \right] \\
= \frac{5}{2}\pi \cos^3 \alpha_3 \left[ 2\int_0^\pi d\vartheta A(\vartheta)\cos^3 \vartheta \sin \vartheta - 3\int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \sin^3 \vartheta \right] \\
- \pi \cos \alpha_3 \left[ \frac{15}{2} \int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \sin^3 \vartheta - 3\int_0^\pi d\vartheta A(\vartheta)\cos \vartheta \sin \vartheta \right]
\]

The program ANNIE performs these integrals numerically for given detector configurations defined by \( r_1, r_2 \) and \( d \) (equation 6). An output and a program listing are at the end of this report. Note that \( K_0 \) is the geometry factor of the telescope and can be analytically calculated (Sullivan, 1968). The values of \( K_0 \) returned by the program are consistent with such calculations. The test case listed on the fourth page of the output is for two detectors separated by a large distance compared to their radii, so that \( A(\vartheta) \) becomes essentially a delta function centered at
\[ \theta = 0. \] Considering the above equations, one can show that the program gives the expected result:

\[ C = \int_{E_1}^{E_2} \frac{dE}{f(E)} \cdot A \Omega \cdot [a_0 + a_1 \cos \alpha_1 + \]

\[ a_2 \frac{3}{2} (\cos^2 \alpha_2 - \frac{1}{3}) + a_3 \frac{5}{2} (\cos^3 \alpha_3 - \frac{3}{5} \cos \alpha_3)] \]

where \( A \Omega = K_0 = 9.87 \times 10^{-6} \). Note the similarity in form between this equation and equation 3.
References


**Nominal L1-L2**

\[
\begin{align*}
R_1 &= 0.9500 \\
R_2 &= 0.9500 \\
D &= 4.080 \\
\theta_{\text{vac}} &= 0.0 \\
\theta_{\text{am}} &= 0.436 \\
\Delta \theta &= 0.0020
\end{align*}
\]

\[
\begin{align*}
\int (A \sin) &= 0.06950 \\
\int (A \cos^2 \sin) &= 0.06626 \\
\int (A \cos^3 \sin) &= 0.06473
\end{align*}
\]

\[
\begin{align*}
K_0 &= A \omega = 0.4367 \\
K_1 &= \cos(\alpha_1) \cdot 0.4263 \\
K_2 &= \cos^2(\alpha_2) \cdot 0.6091 + -0.2030 \\
K_3 &= \cos^3(\alpha_3) \cdot 0.9432 + \cos(\alpha_3) \cdot -0.5659
\end{align*}
\]

**Nominal Window-L1**

\[
\begin{align*}
R_1 &= 1.7400 \\
R_2 &= 0.9500 \\
D &= 1.550 \\
\theta_{\text{vac}} &= 0.471 \\
\theta_{\text{am}} &= 1.048 \\
\Delta \theta &= 0.0020
\end{align*}
\]

\[
\begin{align*}
\int (A \sin) &= 0.73153 \\
\int (A \cos^2 \sin) &= 0.52847 \\
\int (A \cos^3 \sin) &= 0.45957
\end{align*}
\]

\[
\begin{align*}
K_0 &= A \omega = 4.5964 \\
K_1 &= \cos(\alpha_1) \cdot 3.8744 \\
K_2 &= \cos^2(\alpha_2) \cdot 4.0238 + -1.3412 \\
K_3 &= \cos^3(\alpha_3) \cdot 3.5175 + \cos(\alpha_3) \cdot -2.1102
\end{align*}
\]

**VGR 1 LET A**

\[
\begin{align*}
R_1 &= 0.9505 \\
R_2 &= 0.9484 \\
D &= 4.076 \\
\theta_{\text{vac}} &= 0.001 \\
\theta_{\text{am}} &= 0.436 \\
\Delta \theta &= 0.0020
\end{align*}
\]

\[
\begin{align*}
\int (A \sin) &= 0.06947 \\
\int (A \cos^2 \sin) &= 0.06623 \\
\int (A \cos^3 \sin) &= 0.06470
\end{align*}
\]

\[
\begin{align*}
K_0 &= A \omega = 0.4365 \\
K_1 &= \cos(\alpha_1) \cdot 0.4261 \\
K_2 &= \cos^2(\alpha_2) \cdot 0.6088 + -0.2029 \\
K_3 &= \cos^3(\alpha_3) \cdot 0.9427 + \cos(\alpha_3) \cdot -0.5656
\end{align*}
\]
VGR 1 LET B L1-L2
R1 = 0.9479  R2 = 0.9519  D = 4.081
THETAC = 0.001  THETAM = 0.436  DELTH = 0.0020

INTEGRAL(A SIN) = 0.06945  INTEGRAL(A COS SIN) = 0.06779
INTEGRAL(A COS**2 SIN) = 0.06620  INTEGRAL(A SIN**3) = 0.00324
INTEGRAL(A COS**3 SIN) = 0.06467  INTEGRAL(A COS SIN**3) = 0.00312

K0 = A OMEGA = 0.4363
K1 = COS(ALPHA1) * 0.4259
K2 = COS**2(ALPHA2) * 0.6086 + -0.2029
K3 = COS**3(ALPHA3) * 0.9425 + COS(ALPHA3) * -0.5655

VGR 1 LET C L1-L2
R1 = 0.9487  R2 = 0.9478  D = 4.079
THETAC = 0.000  THETAM = 0.435  DELTH = 0.0020

INTEGRAL(A SIN) = 0.06905  INTEGRAL(A COS SIN) = 0.06741
INTEGRAL(A COS**2 SIN) = 0.06583  INTEGRAL(A SIN**3) = 0.00322
INTEGRAL(A COS**3 SIN) = 0.06431  INTEGRAL(A COS SIN**3) = 0.00309

K0 = A OMEGA = 0.4338
K1 = COS(ALPHA1) * 0.4235
K2 = COS**2(ALPHA2) * 0.6053 + -0.2018
K3 = COS**3(ALPHA3) * 0.9374 + COS(ALPHA3) * -0.5624

VGR 1 LET D L1-L2
R1 = 0.9460  R2 = 0.9466  D = 4.075
THETAC = 0.000  THETAM = 0.435  DELTH = 0.0020

INTEGRAL(A SIN) = 0.06863  INTEGRAL(A COS SIN) = 0.06700
INTEGRAL(A COS**2 SIN) = 0.06544  INTEGRAL(A SIN**3) = 0.00319
INTEGRAL(A COS**3 SIN) = 0.06393  INTEGRAL(A COS SIN**3) = 0.00307

K0 = A OMEGA = 0.4312
K1 = COS(ALPHA1) * 0.4210
K2 = COS**2(ALPHA2) * -0.6017 + -0.2006
K3 = COS**3(ALPHA3) * 0.9320 + COS(ALPHA3) * -0.5592
VGR 2 LET A L1-L2
R1 = 0.9471  R2 = 0.9491  D = 4.075
THETAC = 0.000  THETAM = 0.436  DELTH = 0.0020

INTEGRAL(A SIN) = 0.06913  INTEGRAL(A COS SIN) = 0.06748
INTEGRAL(A COS**2 SIN) = 0.06590  INTEGRAL(A SIN**3) = 0.00323
INTEGRAL(A COS**3 SIN) = 0.06438  INTEGRAL(A COS SIN**3) = 0.00310

KO = A OMEGA = 0.4344
K1 = COS(ALPHA1) * 0.4240
K2 = COS**2(ALPHA2) * 0.6059 + 0.2020
K3 = COS**3(ALPHA3) * 0.9383 + COS(ALPHA3) * -0.5630

VGR 2 LET B L1-L2
R1 = 0.9451  R2 = 0.9483  D = 4.064
THETAC = 0.001  THETAM = 0.436  DELTH = 0.0020

INTEGRAL(A SIN) = 0.06908  INTEGRAL(A COS SIN) = 0.06743
INTEGRAL(A COS**2 SIN) = 0.06585  INTEGRAL(A SIN**3) = 0.00323
INTEGRAL(A COS**3 SIN) = 0.06433  INTEGRAL(A COS SIN**3) = 0.00310

KO = A OMEGA = 0.4340
K1 = COS(ALPHA1) * 0.4237
K2 = COS**2(ALPHA2) * 0.6054 + 0.2018
K3 = COS**3(ALPHA3) * 0.9373 + COS(ALPHA3) * -0.5624

VGR 2 LET C L1-L2
R1 = 0.9502  R2 = 0.9424  D = 4.084
THETAC = 0.002  THETAM = 0.434  DELTH = 0.0020

INTEGRAL(A SIN) = 0.06835  INTEGRAL(A COS SIN) = 0.06674
INTEGRAL(A COS**2 SIN) = 0.06519  INTEGRAL(A SIN**3) = 0.00317
INTEGRAL(A COS**3 SIN) = 0.06369  INTEGRAL(A COS SIN**3) = 0.00304

KO = A OMEGA = 0.4295
K1 = COS(ALPHA1) * 0.4193
K2 = COS**2(ALPHA2) * -0.5994 + 0.1998
K3 = COS**3(ALPHA3) * 0.9287 + COS(ALPHA3) * -0.5572
VGR 2 LET D LI-L2
R1 = 0.9499 R2 = 0.9447 D = 4.061
THETAC = 0.001 THETAM = 0.437 DELTH = 0.0020

INTEGRAL(A SIN) = 0.06934
INTEGRAL(A COS**2 SIN) = 0.06609
INTEGRAL(A COS**3 SIN) = 0.06456
INTEGRAL(A COS SIN) = 0.06768

INTEGRAL(A SIN**3) = 0.00312
INTEGRAL(A COS SIN**3) = 0.00325

KO = A OMEGA = 0.437
K1 = COS(ALPHA1) * 0.4252
K2 = COS**2(ALPHA2) * 0.6075 + -0.2025
K3 = COS**3(ALPHA3) * 0.9405 + COS(ALPHA3) * -0.5643

TEST CASE
R1 = 1.000E 00 R2 = 1.000E 00 D = 1.000E 03
THETAC = 0.0 THETAM = 2.000E-03 DELTH = 1.000E-05

INTEGRAL(A SIN) = 1.571E-06
INTEGRAL(A COS**2 SIN) = 1.571E-06
INTEGRAL(A COS**3 SIN) = 1.571E-06
INTEGRAL(A COS SIN**3) = 1.571E-12

KO = A OMEGA = 5.665E-06
K1 = CCS(ALPHA1) * 5.665E-06
K2 = CCS**2(ALPHA2) * 1.480E-05 + -4.935E-06
K3 = CCS**3(ALPHA3) * 2.467E-05 + CCS(ALPHA3) * -1.480E-05
//ANNIE JOB (98221,CZG,SRL), 'N. GEHRELS', TIME=(2.00)
//EXEC FORTG
//FORT DD *
C ----- READ IN DETECTOR RADII AND SEPARATION
C ----- DIMENSION LABEL(19)
COMMON PI,RS,THETAC,THETAM,R1SQR,R2SQR,RSSGR,DSQR,DR12,DR22
5 READ(5,1111,END=999) LABEL
1111 FORMAT(19A4)
READ(5,1112) R1,R2,D
1112 FORMAT(3F10.0)
C ----- INITIALIZE VARIABLES
C ----- CALL INIT(R1,R2,D)
DELTH=0.002
C ----- OUTPUT THE A HEADER
C ----- WRITE(G,3333) LABEL
3333 FORMAT(///1X19A4)
WRITE(G,3334) R1,R2,D,THETAC,THETAM,DELTH
3334 FORMAT( ' R1 = ',FS.4, ' R2 = ',FS.4,
+ ' D = ',FS.3, ' THETAC = ',FS.3,
+ ' THETAM = ',FS.3, ' DELTH = ',FS.4)
C ----- ZERO INTEGRATION SUMS AND INTEGRATE TERMS
C INVOLVING A(THETA)*POWERS OF COS(THETA) AND SIN(THETA)
C FROM THETA=0 TO THETA=THETAM
C ----- SMS=0, SMCS=0, SMC2S=0, SMS3=0, SMCS3=0, THETA=0,
10 THETA=THETA+DELTH
IF(THETA.GT.THETAM) GOTO 20
ATH=A(THETA)
SINTH=SIN(THETA)
COSTH=COS(THETA)
SMS=SMS+ATH*SINTH
SMCS=SMCS+ATH*COSTH*SINTH
SMC2S=SMC2S+ATH*COSTH**2*SINTH
SMS3=SMS3+ATH*SINTH**3
SMCS3=SMCS3+ATH*COSTH**3*SINTH
SMCS3=SMCS3+ATH*COSTH*SINTH**3
GOTO 10
SCALE BY DELTH

20 SMS=SMS*DELTH
SMCS=SMCS*DELTH
SMC2S=SMC2S*DELTH
SMS3=SMS3*DELTH
SMC3S=SMC3S*DELTH
SMCS3=SMCS3*DELTH
WRITE(6,4444) SMS,SMCS,SMC2S,SMS3,SMC3S,SMCS3

4444 FORMAT(/' INTEGRAL(A SIN)=',F7.5,10X'INTEGRAL(A COS SIN)=',
+ F7.5/' INTEGRAL(A COS**2 SIN)=',F7.5,
+ ' INTEGRAL(A SIN**3)=',F7.5/' INTEGRAL(A COS**3 SIN)=',
+ F7.5,' INTEGRAL(A COS SIN**3)=',F7.5)

CALCULATE THE VARIOUS TERMS

TRMK01=2.*PI*SMS
TRMK11=2.*PI*SMCS
TRMK21=(3.*PI/2.)*(2.*SMC2S-SMS3)
TRMK22=PI*(3.*SMS3/2.-SMS)
TRMK31=5.*PI*(2.*SMC3S-3.*SMCS3)/2.
TRMK32=PI*(15.*SMCS3/2.-3.*SMCS)

OUTPUT THE TERMS

WRITE(6,5555) TRMK01,TRMK11,TRMK21,TRMK22,TRMK31,TRMK32

5555 FORMAT(/' KO= A OMEGA =',F8.4/' K1 = COS(ALPHA1) * ,
+ F8.4/' K2 = COS**2(ALPHA2) * ',F8.4,' + ',F8.4/
+ ' K3 = COS**3(ALPHA3) * ',F8.4,' + COS(ALPHA3) * ',
+ F8.4/))
GOTO 5

SUBROUTINE INIT(R1,R2,D)
COMMON PI,RS,THETAC,THETAM,R1SQR,R2SQR,RSSQR,DSGR,DR12,DR22

THIS SUBROUTINE Initializes VARIABLE THAT ARE NEEDED TO CALCULATED A(THETA)

PI=3.14159
RS=AMIN1(R1,R2)
ABSR12=ABS(R1-R2)
THETAC=ATAN(ABSR12/D)
THETAM=ATAN((R1+R2)/D)
R1SQR=R1*R1
R2SQR=R2*R2
RSSQR=RS*RS
DSQR=D*D
DR12=2*D*R1
DR22=2*D*R2
RETURN
END
FUNCTION A(THETA)
COMMON PI,RS,THETAC,THETAM,R1SQR,R2SQR,RSSQR,DSQR,DR12,DR22

THIS FUNCTION Calculates THE PROJECTED OVERLAP AREA OF THE TWO DETECTORS AS A FUNCTION OF THETA
C -----
IF(THETA.GE.THETAC) GOTO 10
A=PI*RSSGR*COS(THETA)
RETURN
10 IF(THETA.LT.THETAM) GOTO 20
A=0.
RETURN
20 TANTH=TAN(THETA)
PHI12=2.*ARCOS((R1SQR+DSGR*TANTH**2-R2SQR)/(DR12*TANTH))
PHI22=2.*ARCOS((R2SQR+DSGR*TANTH**2-R1SQR)/(DR22*TANTH))
A=COS(THETA)*(R1SQR*(PHI12-SIN(PHI12))/2.+
+ R2SQR*(PHI22-SIN(PHI22))/2.)
RETURN
END

//DATA DD *
NOMINAL L1-L2
0.95 0.95 4.08
NOMINAL WINDOW-L1
1.74 0.95 1.55
VGR 1 LET A L1-L2
0.9505 0.9484 4.076
VGR 1 LET B L1-L2
0.9479 0.9519 4.081
VGR 1 LET C L1-L2
0.9487 0.9478 4.079
VGR 1 LET D L1-L2
0.9460 0.9466 4.075
VGR 2 LET A L1-L2
0.9471 0.9491 4.075
VGR 2 LET B L1-L2
0.9451 0.9483 4.064
VGR 2 LET C L1-L2
0.9502 0.9424 4.084
VGR 2 LET D L1-L2
0.9499 0.9447 4.061

// $