RESOLUTION CHARACTERISTICS OF AN ISOTOPE SPECTROMETER COMPOSED OF SILICON SOLID STATE DETECTORS

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ABSTRACT

A stack of silicon solid state detectors was uesd to resolve the isotopes of elements from Ge ($Z = 32$) to Kr ($Z = 36$). Measured resolution values were compared with predicted theoretical values in order to determine some of the major causes of the mass uncertainty.

INTRODUCTION

Silicon solid state detectors are commonly used in accelerator and space-satellite-borne instrumentation to measure the kinetic energy, mass and the charge of energetic nuclei. A typical instrument includes a stack of several disk-shaped detectors. Nuclei lose energy as they penetrate through the stack of detectors, and from the measured energy loss in each detector, such values as R (the range, how far the nucleus travelled in a detector before it stopped), M (the mass of the nucleus), and Z (the charge of the nucleus) can be determined. To date, instruments developed by Caltech and others have resolved isotopes of nuclei ranging from H ($Z = 1$) to Fe ($Z = 26$) in both accelerator experiments and space. As might be expected (according to the theory), the mass resolution is better for the lighter nuclei ($\sigma m \approx 0.1$ amu for C, N, and O) than for heavier nuclei ($\sigma m \approx 0.25$ amu for Fe.) Here σm is the rms mass resolution measured in atomic mass units. In this research, results of Kr ($Z = 36$) fragmentation experiments, which were conducted at the Lawrence Berkeley Laboratory Bevalac in 1984, are examined to see if mass of the isotopes of nuclei that are as heavy as Kr can be resolved by silicon solid state detectors. Results are compared with predicted resolutions which are calculated from theory in order to determine the important contributions to the mass resolution.

METHOD

Set-up of the accelerator runs is as shown in FIGURE 1. At the CH_2 (polyethelene) target, a beam of krypton ($Z = 36, M = 84$ and ~ 400 MeV/nucleon) was fragmented. The resulting nuclei (of various mass and charge) were then detected at different depths of the detector stack according to their range. The detectors used in this study include one of 0.5 mm thickness, one of 1.7 mm, and five 3 mm thick detectors. To illustate the determination of the mass of a nucleus, consider two detectors in the stack. As the nucleus traverses the first detector (of thickness L), energy loss ΔE is measured, while the second detector measures the residual energy loss E' as the nucleus stops, where the total energy is $E = \Delta E + E'$. Since there is a stack of detectors, some of them can be combined to give variety in thickness of Δ E measuring detector. For a particle of mass M, charge Z, and kinetic energy E, the range can be approximated by the relation

$$R = k\frac{M}{Z^2}\left(\frac{E}{M}\right)^{\alpha} \qquad (1)$$

(k is some constant.)

where α=1.75 for 10$\leq$ E/M $\leq$ 100 MeV per nucleon. We can write a similar equation for the range R - L of the particle after traversing a pathlength L, thus

$$R-L = k\frac{M}{Z^2}\left(\frac{E'}{M}\right)^{\alpha}. \qquad (2)$$

Solving this for the mass M, we get

$$M = \left[\frac{k(E^{\alpha}-E'^{\alpha})}{LZ^2}\right]^{\frac{1}{\alpha-1}}. \qquad (3)$$

But we essentially have two equations and three unknowns (R, Z, and M) and the solution won't be unique. However, knowing that the mass M has to be roughly twice the charge Z, we can introduce another quantity which has information on both Z and M. Call it Z'. Rewriting the mass M as M$\simeq$2Z +ΔM, equations (1) and (2) give

$$Z' = Z\left[1+\frac{\Delta M}{2Z}\right]^{\frac{\alpha-1}{\alpha+1}} = \left[\frac{k}{L2^{\alpha-1}}\right]^{\frac{1}{\alpha+1}}\left[E^{\alpha}-E'^{\alpha}\right]^{\frac{1}{\alpha+1}} \qquad (4)$$

Also, R can be simplified to

$$R = \frac{LE^{\alpha}}{E^{\alpha}-E'^{\alpha}}. \qquad (5)$$

In order to resolve the isotopes, first it was necessary to plot the events using the equations for Z' and R as seen in FIGURE 2, and straighten the tracks (which correspond to various elements and isotopes) by adjusting the value α in the equations. Since α depends slightly on detector thickness, an appropriate value of α had to be found for each thickness. A typical value of α is 1.67. Once this was done, a histogram of M for a given element was plotted by taking events in desired range of Z and using the equation for M. In doing so, a limited range of R was set on events where the tracks were straightest (FIGURE 2) in order to obtain the best mass resolution. As an example, FIGURE 3 shows a mass histogram for Br (Z$\simeq$ 35). Note the peaks due to $\sim$10 different isotopes. Then a program which fits gaussians to peaks by the least squares method was used to calculate the mass resolution. Unfortunately, the program only worked for prominent and clearly separated peaks, which was not always the case. When a resolvable fit to the mass histograms could not be obtained, the mass resolution was estimated by taking the full width at half maximum and dividing by 2.35 (Γ= 2.35σ for gaussians where Γ is the fwhm.) The resulting uncertainty is somewhat greater in these cases. The two methods were found to be consistent within reason .

RESULTS AND DISCUSSION

TABLE shows some of the obtained mass resolution. These values were used in the following analysis.

Since the value M depends on several variables (equation (3)), correspondingly there are various uncertainties due to them which "add up" to give the mass resolution. For variable x_1, the uncertainty on M is

$$\sigma^2_{M(x_1)} = \sigma^2_{x_1}\left(\frac{\partial M}{\partial x_1}\right)^2 . \qquad (6)$$

Furthermore,

$$\sigma^2_{M_{total}} = \sum_i \left(\sigma_{x_i}\frac{\partial M}{\partial x_i}\right)^2 \qquad (7)$$

(assuming independency of each σ_x .)

It is expected that the most important such variable is energy-loss fluctuations, which are variations in the value of ΔE lost by the nuclei in the ΔE detector. The value varies because energy loss is a pure statistical process in that it is determined essentially by counting the number of eletrons that the traversing nucleus collides with. The uncertainty on M due to this factor has been calculated by Spalding (1983, Caltech):

$$\sigma_{M(\Delta E)} = \left(\frac{\alpha}{\alpha-1}\right)\frac{M}{E}\left(\frac{\alpha\sigma_0^2 h(\beta) L}{3\alpha-2}\right)^{\frac{1}{2}}\left[\left(\frac{R}{L}\right)^{3-\frac{2}{\alpha}} - \left(\frac{R}{L}-1\right)^{3-\frac{2}{\alpha}}\right] . \qquad (8)$$

Where R and E are average values of range and kinetic energy respectively for a particular set of events, $h(\beta)$ is defined to be

$$h(\beta) = \gamma^2\left(1-\frac{\beta^2}{2}\right) \qquad (9)$$

and σ_0^2 is $\sigma_0^2 = Z^2(0.135 \text{ MeV})^2/(\text{mm of silicon})$. (10)

The predicted resolution values are calculated from this equation and are compared with experimental values for Kr in FIGURE 4 vs. various thicknesses of ΔE detector. Both sets of values show definite dependency on the ΔE detector thickness. As the thickness varies from 0.5 mm to 12.0 mm, they both decrease significantly. However, at each thickness, there's an apparent difference between the predicted and experimental values indicating that there are other contributions that have to be taken into account.

Now consider some experimental uncertainties. The uncertainty in M due to thickness variations of the ΔE detector is

$$\sigma_{M(L)} = \sigma_L\left[\frac{1}{\alpha-1}\left(\frac{1}{L}\right)\right]M . \qquad (11)$$

One factor that's involved in this is the subtle non-uniformity of the thickness of each detector. Obviously, the more uniform it is, the more accurate and better the results are. This can be minimized by choosing a more limited area of the detectors where particles are allowed to penetrate through. Using the proportinal counter, 1/2, 1/4, and 1/20 of the whole area at the center of the detectors were used to determine the significance of this factor on mass resolution. As it turned out, the resolution does actually get better as we approach the center of the detector. But the effect was the greatest in going from the whole area to 1/4 of the area (indicating that the detectors are relatively flat within the center 1/4 of the whole area.) Therefore it was concluded that this factor probably doesn't have a dominant contribution on the uncertainty of M. Comparing the full and 1/20 area results for Kr with a 9 mm ΔE detector thickness, we find $\sigma_{M(L)} \gtrsim 0.13$ amu for the full detector. This corresponds to $\sigma_L \simeq 0.009$ amu out of 9 mm.

Another element that showed a possibility for a significant contribution is uncertainty in the angle at which nuclei enter the detector. In the equation (3), this was taken to be constant at 5°, the angle of the detector stack to the beam. (In fact, because the particles can scatter, there is a distribution of angles about 5°.) The thickness L that has been considered up till now is in fact a path length T/cosθ (FIGURE 5). Taking this into account, equation (3) can be rewritten as

$$M = \left[\frac{k(E^{\alpha}-E'^{\alpha})}{Z^2 T}\cos\theta\right]^{\frac{1}{\alpha-1}} \qquad (3')$$

The mass uncertainty due to the uncertainty in angle θ is

$$\sigma_{M(\theta)} = \sigma_\theta M\left(\frac{1}{\alpha-1}\tan\theta\right) \qquad (12)$$

We can check the contribution of σ_θ to σ_M by using data from a single run in which the detector stack was oriented at $\theta = 10°$ instead of 5°. Here are the measured mass resolutions at these two angles, for Kr nuclei hitting the whole area of the detector.

$$\text{At } \theta = 5° \quad \sigma_{M_{measured}} = 0.287 \pm 0.002$$

$$\text{At } \theta = 10° \quad \sigma_{M_{measured}} = 0.299 \pm 0.004$$

It was possible to see the significance of the contribution of the angle variations to the uncertainty of M from these two values in the following manner. Recall equation (7).

$$\begin{aligned}\sigma^2_{M_{total}} &= \sum_i\left(\sigma_{x_i}\frac{\partial M}{\partial x_i}\right)^2 \\ &= \sigma^2_{x_1}\left(\frac{\partial M}{\partial x_1}\right)^2 + \sigma^2_{x_2}\left(\frac{\partial M}{\partial x_2}\right)^2 + \cdots \qquad (7') \\ &= \sigma^2_{M(\Delta E)} + \sigma^2_{M(\theta)} + \cdots\end{aligned}$$

Then, if in fact $\sigma_{M(\theta)}$ is a dominating factor, it should roughly be equal to

$$\sigma_{M(\theta)} = \sigma_\theta M\left(\frac{1}{\alpha-1}\tan\theta\right) \approx \left(\sigma^2_{M_{measured}} - \underset{(calculated)}{\sigma^2_{M(\Delta E)}}\right)^{\frac{1}{2}}. \qquad (13)$$

Under this assumption, σ_θ of equation (12) can be calculated for both angles and they should match within reason. These calculations gave $\sigma_\theta = 1.24°$ for $\theta = 5°$ and $\sigma_\theta = 0.65°$ for $\theta = 10°$, which are not consistent. On the other hand, we were able to set limits on the value of $\sigma_{M(\theta)}$. Knowing that the angle contribution must be be twice as large at $\theta = 10°$ as at $\theta = 5°$ (from equation (12)), maximum contribution to the data taken at $\theta = 5°$ is calculated to be $\sigma_{M(\theta)} = 0.11$ amu (for Kr data.)

According to theory, the mass resolution should be better for smaller Z. However, the present results show somewhat poorer resolution for $Z = 32$, 34 and 35 nuclei than for $Z = 36$. (See TABLE 1.) It is possible that this is due in part to the fact that the angular variation is greater for fragments than Kr nuclei, since they are scattered at some small angles as Kr beam breaks up (where as unfragmented Kr nuclei would hit the detector perpendicularly.)

SUMMARY AND CONCLUSIONS

Results clearly show that silicon solid state detectors can actually resolve isotopes of element that are as heavy as Kr ($Z = 36$). The measured mass resolution varied from 0.24 to 0.85 amu, depending on the ΔE detector thickness and the fraction of its area used.

Several factors were considered and analyzed as possible major contributions to the mass resolution. The contribution from energy-loss fluctuations proved to be important in that predicted uncertainty from it always took a significant part of the measured resolution and as the graph showed the predicted and experimental values behaved in the same manner for varying thickness of , the ΔE detector. Unlike other factors, energy-loss fluctuations is something that is unavoidable and its corresponding contribution is always present.

Thickness variations in the ΔE detector were tested as we measured resolution with a different sized area of the detector. The mass resolution got better for smaller area, but we found out that there is a limit as to how much we can decrease the area and still obtain a significant decrease in mass uncertainty.

Also, the contribution from angle variations was considered. Measured values at two different angles were used along with the predicted values from energy-loss fluctuations to determine the maximum value of uncertainty, but calculations also showed that its contribution cannot account for a major portion of the mass uncertainty.

In the future, the experimental set-up can be improved for better mass resolution. One suggestion would be to make accurate measurements of the angle of each individual nucleus so that this variation could be eliminated. This would help to improve the mass resolution.

Adding up all the contributing uncertainties that are known or can be estimated at this moment (for Kr at $\theta = 5°$ and 9 mm)

$$\Delta M(\Delta E) = 0.162$$

$$\Delta M(L) \geq 0.132$$

$$\Delta M(\Theta) \leq 0.107$$

$$\Delta M_{total} = 0.235$$

and comparing it with the measured resolution, $\Delta M_{measured} = 0.287$

$$\Delta M_{unexplained} = 0.165$$

(subtract in quadrature).

There is a remaining difference of amu between them that is unaccounted for. This could be due to some other processes that are involved in the detector. One possible process is the electron pick-up. By picking up one or more electrons, nuclei can change their charge and result in energy loss values that are smaller than expected. This process is expected to be more important for heavier nuclei. To investigate this would require studies of a wide range of nuclei at several energies and detector thicknesses.

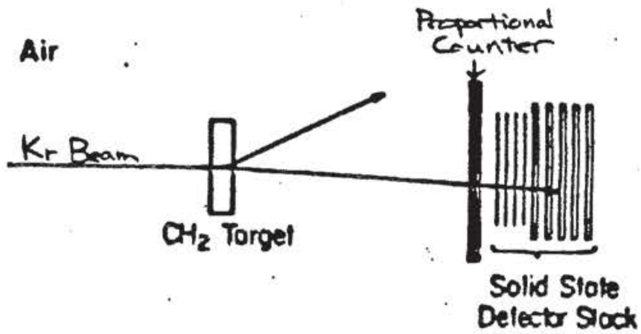


FIGURE 1
Experimental Set-up

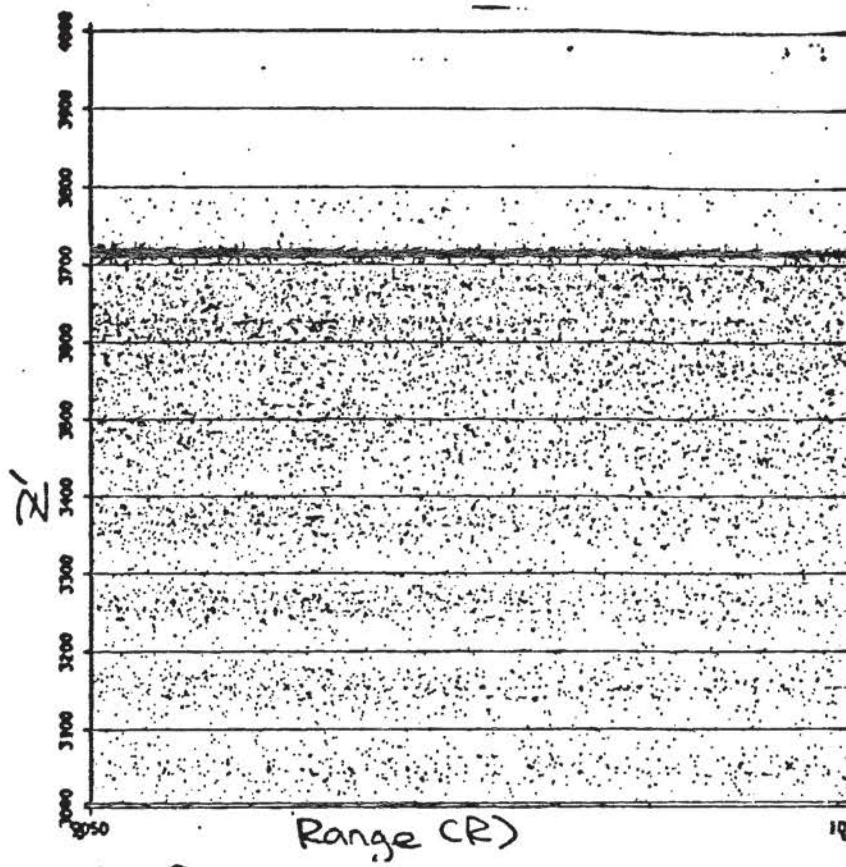


FIGURE 2
Plot of Events (Z' vs R)

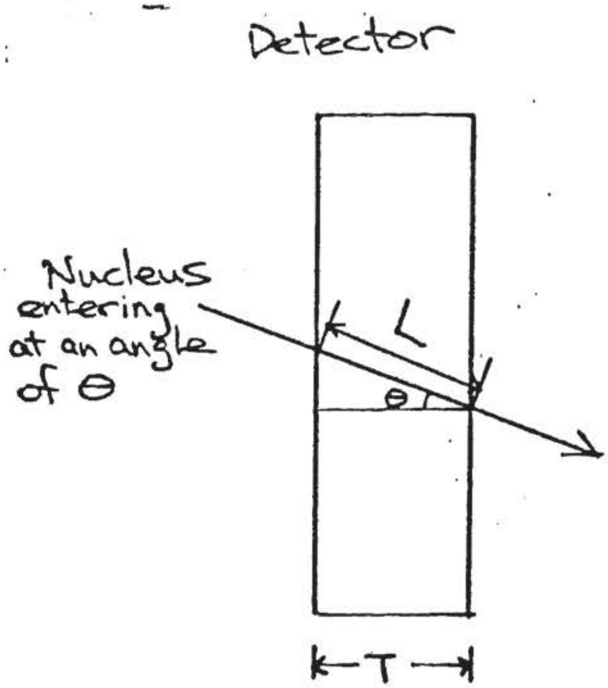


FIGURE 5
Path Length $L = T/\cos\theta$

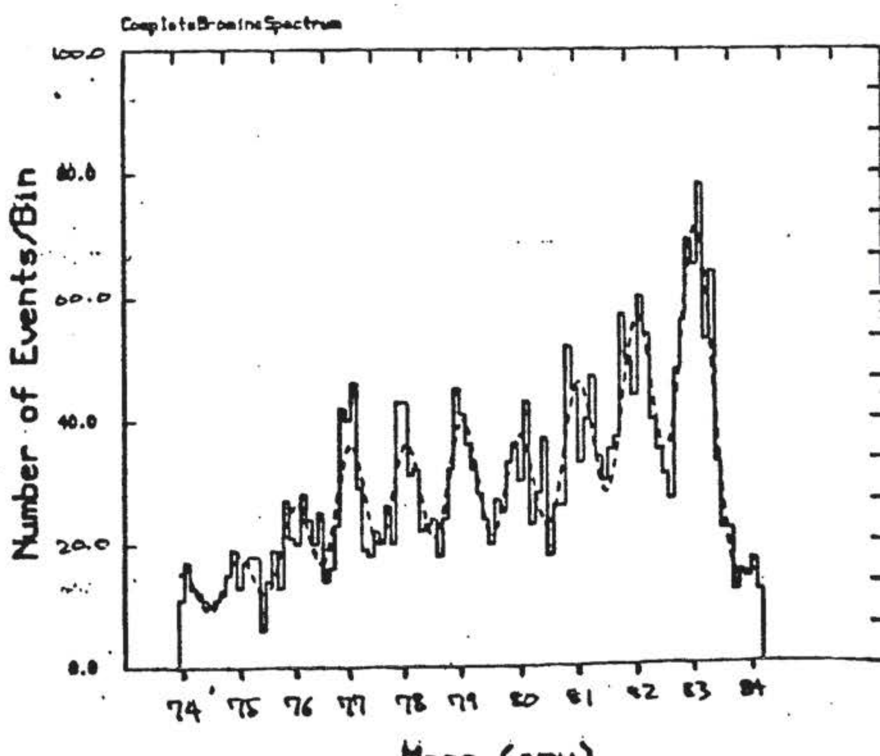
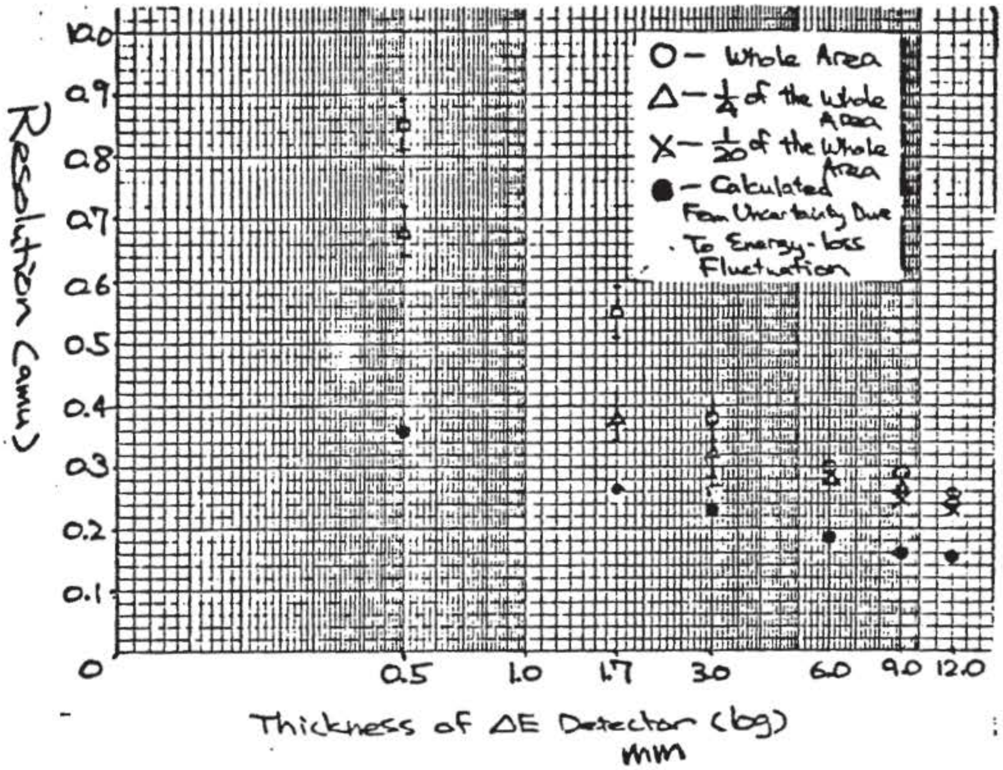


FIGURE 3
Mass Histogram (Isotopes of Br)



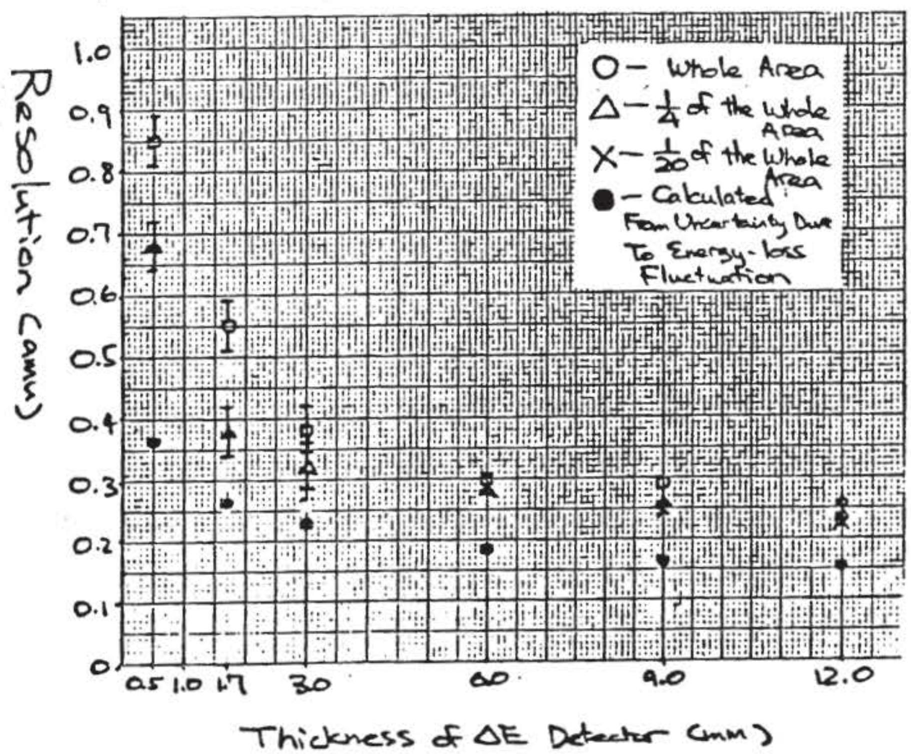


FIGURE 4
Experimental and Predicted Mass Resolution Values For Various Thicknesses of ΔE Detector

TABLE - Mass Resolution Results

Element	ΔE Detector Thickness (mm)	Area of Detector	Based on Fit	Based on fwhm	Predicted*
Kr (Z = 36)	12.0	full	0.256±.002	0.26±.04	0.157
		1/4	0.240±.003	0.26±.04	
		1/20	0.239±.005		
	9.0	full	0.287±.002	0.30±.04	0.162
		1/4	0.260±.002	0.26±.04	
		1/20	0.255±.004		
	6.0	full	0.297±.002	0.30±.04	0.186
		1/4	0.279±.002	0.26±.04	
	3.0	full		0.38±.04	0.231
		1/4		0.34±.04	
	1.7	full		0.55±.04	0.260
		1/4		0.38±.04	
	0.5	full		0.85±.04	0.363
		1/4		0.68±.04	
Br (Z = 35)	12.0	full	0.310±.019		0.153
	9.0	full	0.306±.008		0.158
Se (Z = 34)	9.0	full	0.312±.010		0.158
Ge (Z = 32)	12.0	full	0.283±.021		0.147
	9.0	full	0.302±.013		0.150

* Contribution due to energy-loss fluctuations alone (equation